B14.

136.130: Test #2 Solutions

1. Find A^{-1} or show that the matrix A is not invertible.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \overset{(-1)R_3 \to R_2}{\to} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \overset{(-1)R_2 \to R_1}{\to} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right].$$

So, the inverse of A is $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.

2. Consider the following system of linear equations.

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 1 \\ & & 2y & - & z & = & 2 \\ & & & & z & = & 3 \end{array}$$

Identify the coefficient matrix and write the system in the matrix form (identify clearly the matrices you are using.).

Solution. The coefficient matrix is $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ and the system in matrix form is $A\mathbf{x} = \mathbf{b}$

where $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

3. Compute the determinants of the following matrices. (Hint: use shortcuts based on the theory covered in class.)

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 2 & 3 \\ 3 & 4 & 3 & 4 \\ 4 & 5 & 4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & 7 & 0 & 7 \\ 0 & 1 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 3 & 9 \\ 0 & 8 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

Solution. $\det(A) = 0$ since two columns are equal. $\det(B) = 0$ since there is a 0-column. $\det(C) = (2)(8)(-1) = -16$ since it is an upper triangular matrix.