B13.

136.130: Test #1
Solutions

1. Use **Gauss-Jordan elimination** to solve the following system. Show your work describing your steps. State clearly your final answer.

$$\begin{array}{rcrcrcr} x & + & y & - & z & = & 0 \\ & & 2y & - & 2z & = & -2 \\ & & -y & + & z & = & 1 \end{array}$$

Solution. Start with the augmented matrix then row reduce up to RRE form:

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & -2 & -2 \\ 0 & -1 & 1 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 \text{ to } R_3} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-R_2 \text{ to } R_1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The system associated to the last RRE form matrix is $\begin{array}{rcrcrcr} x & & & & & = & 1 \\ & & y & - & z & = & -1 \\ & & & & 0 & = & 0 \end{array}$. Denoting $z = t$ we have that x=1, y=-1+t, z=t (t ranges through all numbers) is the solution of the original system.

2. Write down one inconsistent linear system with two equations and with unknowns being x and y. Do **not** justify your answer (just write it).

Solution: many. For example $\begin{array}{l} x + y = 1 \\ x + y = 2 \end{array}$ is one such example.

3. We are given

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix},\ B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & -3 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

Perform the operation if possible or indicate it is not possible.

(a) $(3A)(2B)$
(b) $(CB)+A$
(c) $(-2)B - C^T$

Solution. (a) $(3A)(2B)$ can not be done since the sizes of $3A$ and $2B$ are incompatible.

(b) $(CB)+A = \begin{bmatrix} 1 & -3 & 2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}.$

(c) $(-2)B - C^T = (-2)\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 2 \\ 1 & -1 & 0 \end{bmatrix}^T = \begin{bmatrix} -2 & -2 \\ 0 & -2 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -3 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ 3 & -1 \\ 0 & -4 \end{bmatrix}.$