B05.

136.130: Test #5 Solutions

1. Suppose $\mathbf{u} = (1, -2, 3, 0, -1)$ and $\mathbf{v} = (-2, 1, 0, 1, 0)$.

(a) Compute $2\mathbf{u} - 3\mathbf{v}$ and the distance between the points $(1, -2, 3, 0, -1)$ and $(-2, 1, 0, 1, 0)$.

(b) Are **u** and **v** orthogonal? Justify your answer.

Solution. **(a)** $2\mathbf{u} - 3\mathbf{v} = (2, -4, 6, 0, -2) - (-6, 3, 0, 3, 0) = (8, -7, 6, -3, -2)$, and the distance between the twp points is $\sqrt{(1-(-2))^2 + (-2-1)^2 + (3-0)^2 + (0-1)^2 + (-1-0)^2} = \sqrt{29}$.

(b) $\mathbf{u} \cdot \mathbf{v} = (-2) + (-2) + 0 + 0 + 0 = -4$ and since this is not 0, the vectors are not orthogonal.

2. Consider the set **W** containing only the vectors with positive components. Is **W** is a subspace of the vector space $\mathbb{R}^3$? Justify your answer.

Solution. Since (1,2,3) is in **W**, yet (-2)(1,2,3)=(-2,-4,-6) is obviously not in **W**, the set **W** is not closed under scalar multiplication, and so it is not a subspace of $\mathbb{R}^3$.

3. Show that the set $\{(1,0),(0,-1)\}$ of vectors in $\mathbb{R}^2$ spans all of $\mathbb{R}^2$.

Solution. Take any (a,b) in $\mathbb{R}^2$. Then it is easy to see that $(a,b) = a(1,0) + (-b)(0,-1)$ and so each vector in $\mathbb{R}^2$ is a linear combination of the vectors in $\{(1,0),(0,-1)\}$. So, the set $\{(1,0),(0,-1)\}$ spans $\mathbb{R}^2$.