m = {{1, 1}, {1, 0}}

$$\{\{1, 1\}, \{1, 0\}\}$$

Eigenvalues[m]

$$\left\{\frac{1}{2}\left(1+\sqrt{5}\right), \frac{1}{2}\left(1-\sqrt{5}\right)\right\}$$

Eigenvectors[m]

$$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), 1\right\}, \left\{\frac{1}{2}\left(1-\sqrt{5}\right), 1\right\}\right\}$$

p1 = {{1/2 (1 + Sqrt[5]), 1}, {1/2 (1 - Sqrt[5]), 1}}

$$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), 1\right\}, \left\{\frac{1}{2}\left(1-\sqrt{5}\right), 1\right\}\right\}$$

p = Transpose[p1]

$$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), \frac{1}{2}\left(1-\sqrt{5}\right)\right\}, \{1, 1\}\right\}$$

Inverse[p]

$$\left\{\left\{\frac{1}{\sqrt{5}}, \frac{-1+\sqrt{5}}{2\sqrt{5}}\right\}, \left\{-\frac{1}{\sqrt{5}}, \frac{1+\sqrt{5}}{2\sqrt{5}}\right\}\right\}$$

(Inverse[p].m).p

$$\left\{\left\{\frac{1}{\sqrt{5}}+\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}+\frac{-1+\sqrt{5}}{2\sqrt{5}}\right), \frac{1}{\sqrt{5}}+\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}+\frac{-1+\sqrt{5}}{2\sqrt{5}}\right)\right\},\right.$$
$$\left.\left\{-\frac{1}{\sqrt{5}}+\frac{1}{2}\left(1+\sqrt{5}\right)\left(-\frac{1}{\sqrt{5}}+\frac{1+\sqrt{5}}{2\sqrt{5}}\right), -\frac{1}{\sqrt{5}}+\frac{1}{2}\left(1-\sqrt{5}\right)\left(-\frac{1}{\sqrt{5}}+\frac{1+\sqrt{5}}{2\sqrt{5}}\right)\right\}\right\}$$

Simplify[%]

$$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), 0\right\}, \left\{0, \frac{1}{2}\left(1-\sqrt{5}\right)\right\}\right\}$$

d = {{1/2 (1 + Sqrt[5]), 0}, {0, 1/2 (1 - Sqrt[5])}}

$$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), 0\right\}, \left\{0, \frac{1}{2}\left(1-\sqrt{5}\right)\right\}\right\}$$

MatrixPower[d, k]

$$\left\{\left\{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{k}, 0\right\}, \left\{0, \left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^{k}\right\}\right\}$$

```
(p.MatrixPower[d, k]).Inverse[p]
```

$$\left\{\left\{-\frac{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^{1+k}}{\sqrt{5}}+\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{-1-k}}{\sqrt{5}},\right.\right.$$
$$\left.\frac{2^{-2-k}\left(1-\sqrt{5}\right)^{1+k}\left(1+\sqrt{5}\right)}{\sqrt{5}}+\frac{2^{-2-k}\left(-1+\sqrt{5}\right)\left(1+\sqrt{5}\right)^{1+k}}{\sqrt{5}}\right\},$$
$$\left\{-\frac{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^{k}}{\sqrt{5}}+\frac{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{k}}{\sqrt{5}},\ \frac{2^{-1-k}\left(1-\sqrt{5}\right)^{k}\left(1+\sqrt{5}\right)}{\sqrt{5}}+\frac{2^{-1-k}\left(-1+\sqrt{5}\right)\left(1+\sqrt{5}\right)^{k}}{\sqrt{5}}\right\}\right\}$$

```
Simplify[%]
```

$$\left\{\left\{\frac{2^{-1-k}\left(-\left(1-\sqrt{5}\right)^{1+k}+\left(1+\sqrt{5}\right)^{1+k}\right)}{\sqrt{5}},\ \frac{2^{-k}\left(-\left(1-\sqrt{5}\right)^{k}+\left(1+\sqrt{5}\right)^{k}\right)}{\sqrt{5}}\right\},\right.$$
$$\left\{\frac{-\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^{k}+\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{k}}{\sqrt{5}},\ \frac{2^{-1-k}\left(\left(1-\sqrt{5}\right)^{k}\left(1+\sqrt{5}\right)+\left(-1+\sqrt{5}\right)\left(1+\sqrt{5}\right)^{k}\right)}{\sqrt{5}}\right\}\right\}$$

```
ma[k_] := {{(2^(-1-k) (-(1-Sqrt[5])^(1+k)+(1+Sqrt[5])^(1+k)))/Sqrt[5], (2^(-k) (-(1-Sqrt[5])^k+(1+Sqrt[5])^k))/Sqrt[5]},
  {(-((1/2 (1-Sqrt[5]))^k)+(1/2 (1+Sqrt[5]))^k)/Sqrt[5], 1/Sqrt[5]
   2^(-1-k) ((1-Sqrt[5])^k (1+Sqrt[5])+(-1+Sqrt[5]) (1+Sqrt[5])^k)}}
```

```
Simplify[ma[2].Transpose[{{1, 1}}]]
```

```
{{3}, {2}}
```

```
Simplify[ma[5].Transpose[{{1, 1}}]]
```

```
{{13}, {8}}
```

```
Simplify[ma[k - 1].Transpose[{{1, 1}}]]
```

$$\left\{\left\{\frac{2^{-1-k}\left(-\left(1-\sqrt{5}\right)^{1+k}+\left(1+\sqrt{5}\right)^{1+k}\right)}{\sqrt{5}}\right\},\ \left\{-\frac{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{k}\left(-1+\left(\frac{1}{2}\left(-3+\sqrt{5}\right)\right)^{k}\right)}{\sqrt{5}}\right\}\right\}$$