B13.

MATH 1300: Test #5 (Fall 2012)

Solutions & marking scheme:

[7] 1. Let $\mathbf{u} = (1,2,-1,1)$ and $\mathbf{v} = (2,-1,2,0)$.

(a) Compute $(\mathbf{u}\cdot\mathbf{v})(\mathbf{u}+\mathbf{v})$.

(b) Find a vector **w** that is orthogonal to **u.**

Solution. (a) $\mathbf{u}\cdot\mathbf{v}(\mathbf{u}+\mathbf{v}) = (2-2-2)(3,1,1,1) = (-6,-2,-2,-2)$.

(b) For example $\mathbf{w} = (2,-1,0,0)$ is such.

[9] 2. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be the rotation around the origin through the angle of 45° (or $\frac{\pi}{4}$ radians).

(a) Write down the standard matrix corresponding to T.

(b) Find $T(2,7)$.

Solution. (a) The standard matrix corresponding to T is

$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \frac{\sqrt{2}}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

(b) $\frac{\sqrt{2}}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 7 \end{bmatrix} = \frac{\sqrt{2}}{2}\begin{bmatrix} -5 \\ 9 \end{bmatrix}$. So $T(2,7) = \frac{\sqrt{2}}{2}(-5,9)$.

[9] 3. The matrix of the linear transformation $T_1:\mathbb{R}^2 \to \mathbb{R}^2$ is $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$, and the matrix of the linear transformation $T_2:\mathbb{R}^2 \to \mathbb{R}^2$ is $\begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}$.

(a) Which of T_1 and T_2 is one-to-one. Justify your answer.

(b) Find the matrix of the linear transformation $T_2 \circ T_1$.

Solution. (a) The matrix $\begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}$ is invertible (since its determinant is not 0), and the matrix $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$ is not invertible (since its determinant is 0). Hence T_2 is one-to-one, and T_1 is not one-to-one.

(b) The matrix corresponding to $T_2 \circ T_1$ is $\begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 6 & 8 \end{bmatrix}$. (Note the order in the product matters.)