B16. **MATH 1300: Test #4 (Fall 2012)**

Solutions & marking scheme:

[8] 1. Let $\mathbf{u} = (1,2,-1)$ and $\mathbf{v} = (2,-1,2)$.

(a) Compute $\mathbf{u}\cdot\mathbf{v}$.

(b) Find $\cos\theta$, where θ is the angle between the two vectors.

Solution. (a) $\mathbf{u}\cdot\mathbf{v} = 2-2-2 = -2$.

(b) $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \dfrac{-2}{\sqrt{6}\sqrt{9}}$.

[7] 2. Find the volume of the parallelepiped having $\mathbf{u} = (1,2,1)$, $\mathbf{v} = (-1,2,0)$ and $\mathbf{w} = (0,1,1)$ as three adjacent edges.

Solution. The volume is $|\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|$ which in turn is $\begin{vmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 2-1+2 = 3$.

[10] 3. Consider the line l defined by $x = 2t-1, y = t+1, z = -t$, $t \in \mathbb{R}$, and the plane Σ defined by $x - y + z + 3 = 0$.

(a) Are the line and the plane parallel? Why?

(b) If l and Σ are parallel, then compute the distance between the two.

Solution. (a) $\mathbf{v} = (2,1,-1)$ is a vector parallel to l, and $\mathbf{n} = (1,-1,1)$ is orthogonal to Σ. Since $\mathbf{v}\cdot\mathbf{n} = 0$ the two vectors are orthogonal. Hence l is indeed parallel to Σ.

(Alternative: show that the system defined by all equations for l and Σ is inconsistent.)

(b) $(-1,1,0)$ is a point on l. The distance from that point to Σ is $\dfrac{|-1-1+0+3|}{\sqrt{1+1+1}} = \dfrac{1}{\sqrt{3}}$.