

B15.

MATH 1300: Test #3 (Fall 2012)**Solution & marking scheme:**

[8] 1. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and let $B = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ Find

(a) Compute $\det A^{-1}$.

(b) Compute $\det(A+B)$.

Solution. (a) $\det(A) = 2$ and so $\det A^{-1} = \frac{1}{2}$.

(b) $A+B = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$ and $\det(A+B) = 30$.

[9] 2. Suppose A, B and C are 3×3 matrices such that $\det A = 4$, $\det C = 5$, and such that $2AB = C$. Find $\det B$.

Solution. $\det(2AB) = \det C$, so $2^3(\det A)(\det B) = \det C$, and so $\det B = \frac{\det C}{8 \det A} = \frac{5}{32}$.

[8] 3. Let $\mathbf{u} = (3, 0, 4)$ and $\mathbf{v} = (1, 2, -1)$.

(a) Compute $2\mathbf{u} - 3\mathbf{v}$.

(b) Find the coordinates of the unit vector in the direction of \mathbf{u} .

Solution. (a) $2\mathbf{u} - 3\mathbf{v} = (6, 0, 8) - (3, 6, -3) = (3, -6, 11)$.

$\|\mathbf{u}\| = \sqrt{3^2 + 4^2} = 5$ and so $\frac{1}{5}(3, 0, 4) = \left(\frac{3}{5}, 0, \frac{4}{5}\right)$ is the desired vector.