B14. **MATH 1300: Test #2 (Fall 2012)**

Solution & marking scheme:

[8] 1. We row-reduce the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ to the identity matrix as follows:

$\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Find elementary matrices E_1, E_2 and E_3 such that $E_3E_2E_1A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Solution. The first row operation above corresponds to the elementary matrix $E_1 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, the second row operation corresponds to $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, and the third row operation corresponds to $E_3 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$. According to our theory we have $E_3E_2E_1A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ as required.

[9] 2. Find all numbers k such that the matrix $\begin{bmatrix} 1 & 0 \\ 1 & k-2 \end{bmatrix}$ is invertible. Justify your answer.

Solution. A square matrix is invertible if and only if its RREF is the identity matrix. Reducing the given matrix a bit yields $\begin{bmatrix} 1 & 0 \\ 1 & k-2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 \\ 0 & k-2 \end{bmatrix}$. If $k-2 \neq 0$ then multiplying the second row by $1/_{k-2}$ will produce the identity matrix. So, if $k \neq 2$ then the matrix is invertible. On the other hand, if $k = 2$ then the matrix $\begin{bmatrix} 1 & 0 \\ 0 & k-2 \end{bmatrix}$ is already in RREF, and since it is not the identity matrix, the original matrix in that case is not invertible.

[8] 3. Compute $\det(A)$ if (a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, (b) $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 \\ 1 & 2 & 2 & 2 & 2 \end{bmatrix}$.

Solution. (a) using whatever method gives $\det(A) = 1$. (b) This is a lower triangular matrix, and so its determinant is the product of the entries on the main diagonal, which comes to 16.