B13.

MATH 1300: Test #1 (Fall 2012)

Solution & marking scheme:

[8] 1. Find the reduced row echelon form the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$.

Solution. $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to -2R_2 + R_3} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

[9] 2. Use Gauss elimination to solve the system $\begin{matrix} x + y + z = 3 \\ 2x - y - z = 0 \end{matrix}$.

Solution. The augmented matrix is $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & -1 & 0 \end{bmatrix}$. We row-reduce to REF:

$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & -1 & 0 \end{bmatrix} \xrightarrow{(-2)R_1 \text{ to } R_2} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & -3 & -6 \end{bmatrix} \xrightarrow{(-\frac{1}{3})R_2} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \end{bmatrix}$. The corresponding system is $\begin{matrix} x + y + z = 3 \\ y + z = 2 \end{matrix}$. Solution: $z = t$, $y = 2 - t$, $x = 3 - t - (2 - t) = 1$.

[8] 3. The inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & -1 & 1 \\ 0 & a & -1 \end{bmatrix}$. Compute a and show your argument.

Solution. *Since* $AA^{-1} = I_{3,3}$, it follows that the entries on the second column of AA^{-1} are 0, 1 and 0 (from top to bottom). This gives $(1)(-2) + (2)(-1) + 2a = 0$, $(0) + (1)(-1) + a = 1$ and $0 + (2)(-2) + a = 0$. Any one of these three gives $a = 2$