B07. **MATH 1300: Test #3 (Fall 2010)**

Solution & marking scheme:

[7] 1. Find all values of *k* for which the matrix $A = \begin{bmatrix} k & 0 & 0 \\ 1 & k^2 - 2k & 0 \\ 7 & 10 & k+3 \end{bmatrix}$ is NOT invertible.

Solution. The matrix is not invertible when $\det(A) = 0$. Computing:
$\det(A) = (k+1)(k^2 - 2k)(k+3)$ and this is 0 if *k* is one of $\{0,2,-1,-3\}$.

[8] 2. Suppose *A* is a 3×3 matrix with $\det(A) = 2$. Compute the following:

(a) $\det(A^2)$
(b) $\det(4A^{-1})$
(c) $\det(\det(A)A^T)$

Solution. **(a)** $\det(A^2) = \det(A)\det(A) = 4$
(b) $\det(4A^{-1}) = 4^3 \det(A^{-1}) = 4^3 \dfrac{1}{2}$
(c) $\det(\det(A)A^T) = 2^3 \det(A^T) = 2^4$

[6] 3. Compute $\det(A)$ by row-reducing $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ to an upper triangular matrix. No points will be given if other methods are used.

Solution. $\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} \xrightarrow[=]{-R_1 \text{ to } R_2} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} \xrightarrow[=]{\text{switch } R_2 \text{ and } R_3} - \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{vmatrix}$; so $\det(A) = 1$.

[4] 4. Give the points $P(1,2,3)$ and $Q(4,5,6)$, and the vector $\mathbf{v} = (0,1,2)$, compute $\overrightarrow{PQ} - 2\mathbf{v}$.
Solution. $\overrightarrow{PQ} = (4-1, 5-2, 6-3) = (3,3,3)$; so $\overrightarrow{PQ} - 2\mathbf{v} = (3,3,3) - (0,2,4) = (3,1,-1)$.