B05. **MATH 1300: Test #2 (Fall 2010)**

Solution & marking scheme:

[8] 1. Use row reduction to find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ (You will get no points if other methods are used.)

Solution.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \xrightarrow{(-1)R_2 \text{ to } R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{(-2)R_3 \text{ to } R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -2 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right].$$

[9] 2. **(a)** If $A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, find A.

(b) If A is the matrix in part (a) and B is obtained from A by switching the two rows, find the elementary matrix E such that $EA = B$.

Solution. **(a)** A is the inverse of A^{-1}; using the formula for inverses of 2×2 matrices (or any other method) we find $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

(b) E is obtained from $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ by the same row operation (switching the two rows). So,

$E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

[9] 3. We are given $B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$. **(a)** Compute $\det(B)$ **(b)** $Adj(B) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & b \\ 0 & a & -1 \end{bmatrix}$, where a and b are not known. Find them. **(c)** Find B^{-1} using parts (a) and (b).

Solution. **(a)** Expanding along the first row (or using any other method) we get $\det(B) = 1$.

(b) Computing from B: $a = C_{23} = 1$, $b = C_{32} = -1$. So: $Adj(B) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$.

(c) It follows from (a) and (b) that $B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$.