B05. **MATH 1300: Test #1 (Fall 2010)**

Solution & marking scheme:

[9] 1. Find the row reduced echelon form and a row echelon form of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 2 \\ 0 & 2 & 2 \end{bmatrix}$.

Solution. $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 2 \\ 0 & 2 & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{\frac{1}{2}R_3} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. The matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is in REF. Continuing:

$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{(-2)R_2 \text{ to } R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{(-1)R_3 \text{ to } R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and the last matrix is in RREF.

[9] 2. Use Gauss elimination to solve the system $\begin{array}{r} x + y + z = 3 \\ 2x - y - z = 0 \end{array}$.

Solution. The augmented matrix is $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & -1 & 0 \end{bmatrix}$. We row-reduce to REF:

$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & -1 & 0 \end{bmatrix} \xrightarrow{(-2)R_1 \text{ to } R_2} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & -3 & -6 \end{bmatrix} \xrightarrow{(-\frac{1}{3})R_2} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \end{bmatrix}$. The corresponding system is $\begin{array}{r} x + y + z = 3 \\ y + z = 2 \end{array}$. Solution: $z = t$, $y = 2 - t$, $x = 3 - t - (2 - t) = 1$.

[7] 3. We are given $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$. Perform the following matrix operations, if possible; otherwise explain why it is not possible:

(a) A^2 **(b)** BA **(c)** $2A + AB$.

Solution. (a) is not possible since A is not a square matrix (or, since the numbers of columns in A is not the same to the number of rows in A). BA is also not possible, since the numbers of columns in B is not the same to the number of rows in A). **(c)** $AB = \begin{bmatrix} 2 & -1 & 0 \\ -2 & 2 & -1 \end{bmatrix}$, and

$$2A + AB = \begin{bmatrix} 4 & -1 & 2 \\ -2 & 0 & -3 \end{bmatrix}.$$