B13.

MATH 1300: Quiz #4
Solutions

1. (a) Consider the set *S* of all polynomials of type $ax^2 + (2a-3)x + b$, where *a* and *b* range through the set of all real numbers. Is *S* a subspace of the vector space $I\!P_2$ of all polynomials of degree at most 2? Justify your answer.

(b) Consider the set *W* of all triples of type $(a,0,b)$, where *a* and *b* range through the set of all real numbers. Is *W* a subspace of the Euclidean vector space $\mathbb{R}^3$? Justify your answer.

Solution. **(a)** It is not a subspace. For example, the zero polynomial is not in *S*.

(b) This is a subspace:

- $(a_1,0,b_1)+(a_2,0,b_2)=(a_1+a_2,0,b_1+b_2)$ is in *W*
- $k(a_1,0,b_1)=(ka_1,0,kb_1)$ is in *W*.

2. Let *U* be the subspace of the Euclidean vector space $\mathbb{R}^3$ consisting of all triples of type $(a,b,0)$, where *a* and *b* range through the set of all real numbers, and let $S=\{(2,0,0),(0,5,0)\}$. Show that $span(S)=U$.

Solution. $(a,b,0)=\frac{a}{2}(3,0,0)+\frac{b}{5}(0,5,0)$.

3. Is the subset $S=\{(1,0,0),(0,1,0),(1,2,3)\}$ of vectors in $\mathbb{R}^3$ linearly independent? Justify your answer.

Solution. $k_1(1,0,0)+k_2(0,1,0)+k_3(1,2,3)=(0,0,0)$ reduces to solving the system

$$\begin{aligned} k_1 \qquad\quad +k_3 &= 0 \\ k_2 \quad +2k_3 &= 0 \\ 3k_3 &= 0 \end{aligned}$$

It is obvious that the only solution to that system is $k_1=0,\ k_2=0,\ k_3=0$, and so *S* is linearly independent