B15.

MATH 1300: Quiz #3
Solutions

1. Suppose $\mathbf{u} = (1,2,3)$ and $\mathbf{v} = (-2,1,0)$

(a) Compute $2\mathbf{u} - \mathbf{v}$.
(b) Find the components of a vector $\mathbf{w}$ such that $2\mathbf{w} - \mathbf{v} = \mathbf{u}$.
(c) Find the unit vector in the direction of **v.**
(d) Find any (non-zero) vector **z** that is perpendicular to the vector **u**.
(e) Find the cosine of the angle between the vector **u** and the vector $\mathbf{e} = (1,0,0)$. Do not simplify your answer.

Solution.

(a) $2\mathbf{u} - \mathbf{v} = (2,4,6) - (-2,1,0) = (4,3,6)$.

(b) $2\mathbf{w} - \mathbf{v} = \mathbf{u}$ gives $\mathbf{w} = \frac{1}{2}(\mathbf{u} + \mathbf{v}) = \frac{1}{2}(-1,3,3) = (-\frac{1}{2}, \frac{3}{2}, \frac{3}{2})$.

(c) $\|\mathbf{v}\| = \sqrt{4+1+0} = \sqrt{5}$, and so $\frac{1}{\sqrt{5}}(-2,1,0)$ is the unit vector in the direction of **v**.

(d) $\mathbf{z} = (-2,1,0)$ is one such, since $\mathbf{z} \bullet \mathbf{u} = 0$.

(e) Denote by θ the angle between **u** and **e**. Then we have $\cos\theta = \frac{\mathbf{u} \bullet \mathbf{e}}{\|\mathbf{u}\|\,\|\mathbf{e}\|} = \frac{1}{\sqrt{14}}$.