B14.

MATH 1300: Test #2
Solutions

1. Use row reduction to find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. (No marks will be given if other methods are used.)

Solution. Start with the augmented matrix then row reduce up to RRE form:

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-R_1 \text{ to } R_3} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{-R_2 \text{ to } R_3} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{-R_3 \text{ to } R_2} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

So the inverse of the given matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$.

2. Suppose A is a 3×3 matrix such that $A^2 + 3A = I$ (where, as usual, I is the identity matrix of the same size), and suppose $(A + 3I)^T = B$. Find A^{-1} in terms of B.

Solution. Starting from $A^2 + 3A = I$ and multiplying both sides by A^{-1} (on any side) we get $A + 3I = A^{-1}$. So, $A^{-1} = B^T$.

3. Compute the determinant of $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$.

Solution. Expanding along the third column gives

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \end{vmatrix} = 1(-1)^{1+3}(6-1) + 0 + 1(-1)^{3+3}(1-2) = 4 .$$