B13.

MATH 1300: Test #1
Solutions

1. Use **Gauss-Jordan elimination** to solve the following system. Show your work describing your steps. State clearly your final answer. (No marks will be given if you do not use Gauss-Jordan elimination!)

$$\begin{aligned} x + y - z &= 0 \\ 2y - 2z &= -2 \\ 3z &= 3 \end{aligned}$$

Solution. Start with the augmented matrix then row reduce up to RRE form:

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & -2 & -2 \\ 0 & 0 & 3 & 3 \end{bmatrix} \xrightarrow{\frac{1}{3}R_3} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{2R_3 \text{ to } R_3} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \underset{R_3 \ to \ R_1}{\xrightarrow{(-1R_2 \ to \ R_1}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

The system associated to the last RRE form matrix is $\begin{aligned} x &= 1 \\ y &= 0 \\ z &= 0 \end{aligned}$. So (1,0,0) is the only solutions of the original system.

2. We are given

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & -3 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

Perform the operation if possible or indicate it is not possible. (a) $(3A)(2B)$ (b) $(CB)+A$ (c) $(-2)B - C^T$

Solution. (a) $(3A)(2B)$ cannot be done since the sizes of $3A$ and $2B$ are incompatible.

(b) $(CB)+A = \begin{bmatrix} 1 & -3 & 2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$.

(c) $(-2)B - C^T = (-2)\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 2 \\ 1 & -1 & 0 \end{bmatrix}^T = \begin{bmatrix} -2 & -2 \\ 0 & -2 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -3 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ 3 & -1 \\ 0 & -4 \end{bmatrix}$.

3. Write down the inverse of the given matrix, if the inverse exists. Otherwise state that the matrix is not invertible.

(a) $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ (b) $B = \begin{bmatrix} 6 & 0 \\ 3 & 0 \end{bmatrix}$

Solution: (a) $A^{-1} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}$ (b) B is not invertible.