4.

136.130: Test #4 Solutions

1. Find the rank of the following matrix.

(a) $\begin{matrix} 1 & 0 & 2 \\ 1 & 0 & 2 \\ 1 & 0 & 2 \end{matrix}$

This matrix can obviously be row reduced to $\begin{matrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}$ so that the row reduced echelon form will have one non-zero row. So the rank of the matrix is 1.

(b) $\begin{matrix} 1 & 1 & y \\ 1 & 2 & y \end{matrix}$, where x and y are any numbers.

$\begin{matrix} 1 & 1 & y \\ 1 & 2 & y \end{matrix} \quad \text{(1) 1st to 2nd} \quad \begin{matrix} 1 & 1 & y \\ 0 & 1 & 0 \end{matrix}$ yields the row reduced echelon form. There are 2 non-zero rows, so the rank is 2.

2. Compute the determinants of the following matrices.

(a) $\mathbf{A} = \begin{matrix} 1 & 2 & 1 \\ 3 & 0 & 3 \\ 3 & 2 & 3 \end{matrix}$

$\det \mathbf{A} = \mathbf{0}$, since there are two equal columns.

(b) $\mathbf{B} = \begin{matrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{matrix}$

This is a diagonal matrix and $\det \mathbf{B} = (\ \ 2)(\ \ 2)(\ \ 3) \quad 12$.

(c) $\mathbf{C} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$

Expand along the first row to get $\det C = (\ \ 1)(\ \ 1)^{1+1} \det \begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} = (\ \ 1)$

3. Suppose $\mathbf{A} = \begin{matrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{matrix}$. Show that A is invertible. Find $\det(\mathbf{A}^{\ 1})$.

The matrix is diagonal and $\det \mathbf{A} = \ \ 2$. Since that is not 0, A is invertible. We have $\det(\mathbf{A}^{\ 1}) = \frac{1}{\det \mathbf{A}} = \ \ \frac{1}{2}$.