Name (Print)__________________________________

I understand that cheating is a serious offense.

Signature:__________________________________

Student Number__________________________________

THE UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS

136.130 Vector Geometry and Linear Algebra
Mid-Term Exam

Date: Friday, October 26, 2001
Time: 5:30–6:30 PM

Identify your section

	Section	Instructor	Slot	Time	Room
☐	L01	C R Platt	2	MWF 9:30–10:20	224 Engin.
☐	L02	S Kalajdzievski	8	MWF 1:30–2:20	221 Wallace
☐	L03	B E Stebnicky	8	MWF 1:30–2:30	225 St. Paul's
☐	L04	C K Gupta	9	TT 11:30–1:00	204 Armes
☐	Other	(SJR, deferred, etc.)			

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IN THIS COLUMN

1	/
Total	/0

Instructions

*Give the required information above, including your printed name, your student number, **and your signature**. An exam without a signature could be declared unacceptable!*

This is a one-hour exam.

No calculators, texts, notes, or other aids are permitted.

***Show your work clearly** for full marks.*

This exam has 0 questions on 0 pages, for a total of 0 points.
***Check now** that you have a complete exam.*

*There are two extra blank pages at the end for rough work. **Do not separate the pages.***

*Answer all questions on the exam paper in the space provided. If you need more room, you may continue your answer on the reverse side, but **clearly indicate** that your work is continued there.*

THE UNIVERSITY OF MANITOBA

Friday, October 26, 2001, 5:30–6:30 PM | Mid-Term Exam

COURSE: Mathematics 136.130 Vector Geometry and Linear Algebra | TIME: 1 HOUR

EXAMINER: Gupta, Kalajdzievski, Platt, Stebnicky

[Values]

[10] **1.** Let $\mathbf{u} = (1, -2, 0)$, $\mathbf{v} = (3, 0, -2)$, $\mathbf{w} = (3, 4, -6)$, $\mathbf{p} = (2, 4, -2, 0)$, and $\mathbf{q} = (1, 0, -1, 1)$.

Calculate each of the following.

(a) $2\mathbf{u} + 3(\mathbf{v} - 2\mathbf{u})$

$$2\mathbf{u} + 3(\mathbf{v} - 2\mathbf{u}) = (5, 8, -6)$$

(b) $\big(\mathbf{p} \bullet \mathbf{q}\big)\mathbf{v} \times \mathbf{u}$

$$\big(\mathbf{p} \bullet \mathbf{q}\big)\mathbf{v} \times \mathbf{u} = 4 \cdot (-4, -2, -6) = (-16, -8, -24)$$

(c) The unit vector with the same direction as $\mathbf{q}$.

$$\frac{(1, 0, -1, 1)}{\sqrt{1 + 0 + 1 + 1}} = \frac{(1, 0, -1, 1)}{\sqrt{3}}$$

(d) The cosine of the angle between $\mathbf{u}$ and $\mathbf{w}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{w}}{||\mathbf{u}||\ ||\mathbf{v}||} = \frac{-5}{\sqrt{5}\sqrt{51}}$$

(e) The angle between $\mathbf{u}$ and $\mathbf{w} \times \mathbf{u}$.

$\mathbf{w} \times \mathbf{u}$ is perpendicular to $\mathbf{u}$. So the angle between the two is $\frac{\pi}{2}$.

[Values]

[15] **2.**

(a) Let L be the line containing point $Q(1,1,0)$ and which is parallel to $\mathbf{v} = (1,1,3)$. Find parametric equations for the line L.

The parametric equations can be read straight from what we are given.

$$x = 1 + t$$

$$y = 1 + t$$

$$z = 3t$$

(b) Find the point of intersection of the line L of part **(a)** with the plane Π whose equation is $3x + 2y + z = 2$.

Solve the system defined by the three equations from (a) and the one defining the plane. Get (with $t = -\frac{3}{8}$) $x = \frac{5}{8}$, $y = \frac{5}{8}$ and $z = -\frac{9}{8}$. So, the intersection point is $(\frac{5}{8}, \frac{5}{8}, -\frac{9}{8})$.

(c) Find an equation in standard form of the plane containing the points $P(-1,2,1)$, $Q(1,1,0)$, and the origin, $(0,0,0)$.

$\mathbf{n} = (-1,2,1) \times (1,1,0) = (-1,1,-3)$ is a vector perpendicular to the plane. So, the point normal equation of the plane is

$$((x,y,z) - (1,1,0)) \cdot (-1,1,-3) = 0.$$

After expanding we get the equation in standard form:

$$-x + y - 3z = 0.$$

[Values]

[12] **3.** Given the system

$$\begin{array}{rcrcrcr} 3x_1 & + & 3x_2 & + & 6x_3 & = & 3 \\ 2x_1 & + & x_2 & - & x_3 & = & 3 \\ & - & x_2 & - & 5x_3 & = & 1 \\ -5x_1 & - & 2x_2 & + & 5x_3 & = & -8 \end{array}$$

(a) Write the augmented matrix of this system.

$$\begin{bmatrix} 3 & 3 & 6 & | & 3 \\ 2 & 1 & -1 & | & 3 \\ 0 & -1 & -5 & | & 1 \\ -5 & -2 & 5 & | & -8 \end{bmatrix}$$

(b) Convert the augmented matrix into reduced row echelon form using elementary row operations. Specify which operations you are using.

The row reduced echelon form of the augmented matrix is

$$\begin{bmatrix} 1 & 0 & -3 & | & 2 \\ 0 & 1 & 5 & | & -1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}.$$

(c) Give the general solution of the system, using parameters where necessary.

Set $x_3 = t$ to get $x_1 = 2 + 3t$ and $x_2 = -1 - 5t$. So, the general solution is $(x_1, x_2, x_3) = (2 + 3t, -1 - 5t, t)$.

[Values]

[10] **4.** Let

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 6 & 2 \\ 3 & 3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -4 & -1 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}$$

For each expression, **first** determine **whether it is defined**. If so, simply answer "Yes." If it is not defined, state the reason why not (answer "No, because . . . "). You should be able to do this part without doing any calculation.

Then, If the expression *is* defined, evaluate it. (Note: In some cases, you can use rules for matrix algebra to simplify the expression, or even to determine the answer without calculation.)

(a) $DA - AD$

$$DA - AD = \begin{bmatrix} 2 & -1 \\ -2 & -2 \end{bmatrix}.$$

(b) $(AD)^T - D^T A^T$

By a property linking transpose with matrix multiplication the result is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

(c) $BC - CB$

This is not defined: BC is a 2 by 2 matrix, while CB is 3 by 3 matrix.

(d) $3A^T + BC$

$$3A^T + BC = 3\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 12 & 2 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 18 & 2 \\ 1 & 2 \end{bmatrix}.$$

[Values]

[13] **5.**

(a) Find the inverse matrix A^{-1} of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -2 \\ 1 & 4 & 0 \end{bmatrix}$, **and verify your answer by matrix multiplication**.

The row reduction technique yields $A^{-1} = \begin{bmatrix} -8 & -4 & 1 \\ 2 & 1 & 0 \\ 9 & 4 & -1 \end{bmatrix}$. To check the answer we multiply $A \cdot A^{-1}$ and see if the result is the identity matrix of size 3.

(b) Let $\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. By using A^{-1}, find the solution to $A\mathbf{x} = \mathbf{p}$, and the solution to $A\mathbf{x} = \mathbf{q}$, where A and A^{-1} are as in part **(a)**. No other methods may be used; no points will be given if A^{-1} is not made use of (meaningfully).

The solution of $A\mathbf{x} = \mathbf{p}$ is $\mathbf{x} = A^{-1}\mathbf{p} = \begin{bmatrix} -12 \\ 3 \\ 13 \end{bmatrix}$

The solution of $A\mathbf{x} = \mathbf{q}$ is $\mathbf{x} = A^{-1}\mathbf{q} = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$