THE UNIVERSITY OF MANITOBA

JUNE 15, 2001 **SUMMER EVENING** FINAL EXAMINATION

PAPER NO. 28 PAGE NO: 1 of 10

DEPARTMENT & COURSE NO: 136.130 TIME: 2 HOURS

EXAMINATION: Vector Geometry & Linear Algebra EXAMINER: D. Stebnicky

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NAME:____
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(Please print clearly) (ID number)

SIGNATURE:____
(I understand that cheating is a serious offence)

INSTRUCTIONS TO THE STUDENT

This is a two-hour exam. Please show your work clearly.

No calculators, texts, notes or other aids are permitted.

This exam has a header page, 9 pages of questions and two blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 120.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Values

[10] 1. Consider the following system of equations:

$$\begin{array}{rcrcrcr} x_1 & - & 3x_2 & + & x_3 & = & 1 \\ -2x_1 & + & x_2 & + & 3x_3 & = & -7 \\ x_1 & - & 4x_2 & + & 2x_3 & = & 0 \end{array}$$

(a) Solve the system by transforming the augmented matrix to row reduced echelon form.

(b) Express your solution in vector form and give a geometric interpretation of the intersection of the three planes represented by the equations.

Values

[16] 2. Given $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 1 & 1 \\ 2 & -2 & -1 \end{bmatrix}$

$D = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & 0 \end{bmatrix}$, $E = \begin{bmatrix} 4 & -6 \\ -5/3 & 5/2 \end{bmatrix}$

find the following, or explain why they do not exist:

(a) $2A + B^T D$

(b) E^{-1}

(c) AC

(d) $\mathbf{x}$, the solution to $A\mathbf{x} = \mathbf{b}$ such that $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

Values

[6] 3.

$$\begin{aligned} -x + 2y - 3z &= 1 \\ 2x \phantom{{}+2y} + z &= 0 \\ 3x - 4y - 4z &= 0 \end{aligned}$$

Use Cramer's rule to solve for x.

[8] 4. Find an equation in point-parallel form of the line through $P(2, 1, -2)$ and perpendicular to the plane $3x + 2y + 5z = 7$. What is the point of intersection of this line with the plane $2x - 3y + z = -4$?

Values

[16] 5. Let $P(1,2-1)$, $Q(3,3,-4)$ and $R(2,3,0)$ be three points in R^3.

(a) Find a point-normal form and a standard form for the plane determined by P, Q and R.

(b) Find the area of the triangle determined by P, Q and R.

(c) Find $\cos\theta$ where θ is the angle at P. What can you conclude about $\overline{PQ}$ and $\overline{PR}$?

Values

[8] 6. Consider the system of equations: $\begin{array}{rcrcl} ax_1 & - & 8x_2 & = & 1 \\ -2x_1 & + & ax_2 & = & b \end{array}$.

(a) Using the coefficient matrix of the system find the values of a such that the system will not have a unique solution.

(b) If $a = 4$ find the value of b that will make the system have infinitely many solutions.

[4] 7. For matrices A and B, assume that $A(BA)$ is defined. Let A be of size 4×5. Find the size of B. (explain).

Values

[12] 8. Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Find the adjoint of A.

(b) Using the adjoint of A, find A^{-1}.

Values

[16] 9. (a) Let $\mathbf{v}_1 = (1,-1,0,1)$, $\mathbf{v}_2 = (-2,3,-1,0)$ and $\mathbf{v}_3 = \left(1,-\tfrac{1}{2},-\tfrac{1}{2},2\right)$.
Express $\mathbf{v}_3$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

(b) Determine whether or not the following vectors in R^3 are linearly dependent $\mathbf{v}_1 = (-1,0,1)$ $\mathbf{v}_2 = (4,-3,5)$ $\mathbf{v}_3 = (1,-4,3)$.

Values

[16] 10. Consider the homogeneous system of equations:

$$\begin{aligned} x_1 - 2x_2 - 2x_3 + x_4 &= 0 \\ -2x_1 + 3x_2 - x_3 + 4x_4 &= 0 \\ 4x_1 - 7x_2 - 3x_3 + 6x_4 &= 0 \end{aligned} .$$

(a) Find a basis for the solution space of the system.

(b) Let A be the coefficient matrix of the system. Find a basis for the column space of A such that the basis is made up of columns of A.

(c) What is the rank of the row space of A?

Values

[8] 11. Let S be the set of all vectors of the form $(a, b, 2a + b)$ in R^3.

(a) Prove that S is a subspace of R^3.

(b) Find a spanning set for S.