THE UNIVERSITY OF MANITOBA

April 15, 1999 FINAL EXAMINATION

PAPER NO: 384

DEPARTMENT & COURSE NO: 136.130

TIME: 2 HOURS

EXAMINATION: Vector Geometry & Linear Algebra

EXAMINER: Various

Values

[11] **1.** Let $\mathbf{u} = (1, 2, -2)$ and $\mathbf{v} = (2, 3, 1)$ be two vectors in $\mathbf{R}^3$.

(i) Compute $5\mathbf{u} - 3\mathbf{v}$.

(ii) Compute $\mathbf{u} \times \mathbf{v}$.

(iii) Compute $\mathrm{d}(\mathbf{u}, \mathbf{v})$.

(iv) Find $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} \times \mathbf{v})$

Values

[10] **2.** Let $P(3,1,0)$, $Q(1,2,0)$ and $R(0,1,1)$ be three points in $\mathbf{R}^3$. Find an equation in standard form for the plane that passes through P, Q and R.

Values

[8] **3**. Consider the point $P(6,3,-5)$ and the plane $\Pi: 2x+y-3z=2$ in $\mathbf{R}^3$.

(a) Find parametric equations for the line through P perpendicular to the plane Π.

(b) Find the point of intersection of the line in part (a) with the plane Π.

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Values

[12] **4.** Use matrices and the Gauss-Jordan method of elimination to solve the following system of equations; describe your answer in terms of the parameters $s, t, u, v, \ldots$ (as necessary):

$$\begin{array}{rcrcrcrcrcr} x_1 & + & 3x_2 & - & x_3 & + & x_4 & - & 2x_5 & = & 8 \\ x_1 & + & 6x_2 & - & 4x_3 & + & 2x_4 & - & x_5 & = & 14 \\ 2x_1 & + & 8x_2 & - & 4x_3 & + & 3x_4 & - & 4x_5 & = & 21 \end{array}$$

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Values

[10] **5.** (a) If k is a nonzero scalar and if the square $n \times n$ matrix A has an inverse, show that kA has an inverse and find an expression for this inverse in terms of k and A^{-1}.

(b) Evaluate the determinant of A where $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 1 & 3 & 8 \\ 1 & 2 & -1 & 3 \\ 3 & 1 & -2 & 1 \end{bmatrix}$.

Values

[8] **6**. Use Cramer's rule to solve for x_2 **only**, where

$$\begin{aligned} x_1 - x_2 + x_3 &= 0 \\ x_1 + x_2 - 2x_3 &= 1 \\ x_1 + 2x_2 + x_3 &= 6 \end{aligned}$$

Values

[13] **7.** (a) Find two 2×2 matrices $A \neq 0$ and $B \neq 0$ such that $AB = 0$.

(b) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 8 \\ 3 \end{bmatrix}$. Find the adjoint of A and use it to find A^{-1}. Then use A^{-1} to solve the matrix equation $A\mathbf{x} = \mathbf{b}$.

Values

[12] **8.** (a) Determine which of the following statements are true and which are false. Give reasons for your answers. We will give no marks if you simply state true or false, with no justification.

(i) [A matrix B is symmetric if $B^T = B$]. If A is any $n \times n$ matrix then $A + A^T$ is symmetric (true or false?)

(ii) If A, B and C are $n \times n$ matrices, $A \neq 0$ and $AB = AC$ then $B = C$ (true or false?)

(iii) Suppose that A is an $m \times n$ and B is a $p \times q$ matrix. If AB is defined and square, then BA is also defined and square (true or false?).

(b) For the system $A\mathbf{x} = \mathbf{b}$, suppose that A is a 5×7 matrix, the rank of A is 3 and the rank of $[A|\mathbf{b}]$ is also 3. State (without proof) the nature of the solution(s) (i.e., the number of solutions and the number of parameters).

Values

[12] **9.** For each of the following sets of vectors, determine whether it is linearly dependent or independent; if dependent, then write one of the vectors as a linear combination of the other two:

(a) $\{\mathbf{v}_1 = (1,1,0),\ \mathbf{v}_2 = (2,3,1), \mathbf{v}_3 = (2,3,2)\}$.

(b) $\{\mathbf{v}_1 = (1,0,2,0),\ \mathbf{v}_2 = (2,1,3,2), \mathbf{v}_3 = (7,2,12,4)\}$

Values

[12] **10.** Let S be the set of all vectors (x,y,z) in $\mathbf{R}^3$ such that $z = x + y$.

(a) Prove that S is a subspace of $\mathbf{R}^3$.

(b) Find a spanning set for S.

Values

[12] **11.** (a) Find a basis for the subspace generated by (or spanned by) the given set of vectors in $\mathbf{R}^4$:

$$\{\mathbf{v}_1 = (1,1,0,0), \quad \mathbf{v}_2 = (0,1,0,0), \quad \mathbf{v}_3 = (2,5,0,0), \quad \mathbf{v}_4 = (0,1,1,2), \quad \mathbf{v}_5 = (1,5,2,4)\}$$

Note: also do part (b) below.

(b) What is the dimension of the subspace in part (a)? Justify your answer.

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