B02.

MATH 1200: Test #5 (Fall 2011)

Solution; marking scheme

[4] 1. [2] (a) Convert 35º in radians.
[2] (b) Convert 2.5 radians in degrees.

Solution.

(a) 35º is $35\dfrac{\pi}{180}$ radians. (This is 0.61 radians.)

(b) 2.5 radians is $2.5\dfrac{180}{\pi}$ degrees, or 143.24º.

[6] 2. Find all solutions of $\cos 4\theta = \dfrac{1}{2}$.

Solution. Set $\alpha = 4\theta$; $\cos\alpha = \dfrac{1}{2}$ gives $\alpha = \dfrac{\pi}{3} + 2k\pi$ or $\alpha = -\dfrac{\pi}{3} + 2k\pi$, where k ranges through all integers. Hence $4\theta = \dfrac{\pi}{3} + 2k\pi$ or $4\theta = -\dfrac{\pi}{3} + 2k\pi$, from where it follows that $\theta = \dfrac{\pi}{12} + k\dfrac{\pi}{2}$ or $\theta = -\dfrac{\pi}{12} + k\dfrac{\pi}{2}$, where k moves through all integers.

[5] 3. [2] (a) Express $z = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i$ in polar form.

[3] (b) Use your answer in (a) to find z^{14}; your final answer should be as simple as possible and in Cartesian coordinates.

Solution. (a) $z = \cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}$.

(b) $z^{14} = \left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)^{14} = \cos\dfrac{14\pi}{4} + i\sin\dfrac{14\pi}{4} = \cos\dfrac{7\pi}{2} + i\sin\dfrac{7\pi}{2} = -i$