B04.

MATH 1200: Test #4 (Fall 2011)

Solution; marking scheme

Important: In all three questions use the function $f(x) = x^3 - 3x + 1$ **.**

[7] 1. [3] (a) Show that there is a solution *c* of $f(x) = 0$ such that $0 < c < 1$.

[4] (b) The first two steps of approximating the solution $f(x) = 0$ that is between 0 and 1 by means of the bisection method are given in the table below. Continue the procedure one more step and complete the third row of the table.

n	a	x_n	b	$f(a)$	$f(x_n)$	$f(b)$
1	0	0.5	1	1	-0.375	-1
2	0	0.25	0.5	1	0.265	-0.375
3						

Solution.
(a) As seen from the table, $f(0)f(1) = (1)(-1) < 0$, and the result follows from the Intermediate Value Theorem.
(b)

3	0.25	0.375	0.5	0.265	-0.072	-0.375

[4] 2. There is a solution *d* of $f(x) = 0$ such that $1 < d < 2$. What is the first approximation x_1 of the solution *d* if the method of linear interpolation is being used on the interval from 1 to 2?

Solution. $x_1 = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = \dfrac{1f(2) - 2f(1)}{f(2) - f(1)} = 1.25$

[4] 3. The equation $f(x) = 0$ has the same set of solutions as the rearrangement $x = \dfrac{x^3 + 1}{3}$. The starting approximation is $x_1 = 0$. Use the method of successive approximations and the rearrangement $x = \dfrac{x^3 + 1}{3}$ to compute the next two approximations x_2 and x_3 of a solution of $f(x) = 0$.

Solution. $x_2 = \dfrac{x_1^3 + 1}{3} = \dfrac{1}{3} = 0.333$; $x_3 = \dfrac{x_2^3 + 1}{3} = \dfrac{(0.333)^3 + 1}{3} = 0.3456$.