B03. **MATH 1200: Test #3 (Fall 2011)**

Solution; marking scheme

[4] 1. For $z_1 = 2+i$ and $z_2 = 1-i$ write the following in the Cartesian form.

[3] (a) $z_1 z_2$

[2] (b) $\dfrac{\overline{z_1}+1}{z_2}$

Solution.

(a) $z_1 z_2 = (2+i)(1-i) = 2+i-2i+1 = 3-i$

(b) $\dfrac{\overline{z_1}+1}{z_2} = \dfrac{(2-i)+1}{1-i} = \dfrac{3-i}{1-i}\dfrac{1+i}{1+i} = \dfrac{4-2i}{2} = 2-i$.

[3] 2. Compute the remainder if $x^5 - 2x^4 + x - 3$ is divided by $x-1$?

Solution. Denoting $P_5(x) = x^5 - 2x^4 + x - 3$, by the theorem in class (and book) the remainder is $P_5(1) = -3$.

[7] 3. Consider the polynomial $P_6 = x^6 + x^5 - x^3 + x - 6$.

[4] (a) Use the Descartes' Rules of Signs to find the numbers of possible positive and negative real zeros of $P_6(x)$.

[3] (b) Use he rational root theorem to list all possible rational zeros of $P_6(x)$. (Do NOT check if they are zeros; just list them.)

Solution. (a) There are 3 changes of signs in $P_6(x)$; so the number of possible positive real zeros is 3 or 1. Compute $P_6(-x) = x^6 - x^5 + x^3 - x - 6$; there are also 3 changes of signs of coefficients. So, the number of possible negative real zeros is also 3 or 1.

(b) Looking at all solutions of type $\dfrac{p}{q}$, rational in lowest terms. Since $a_6 = 1$ we may take $q = 1$. Since $a_0 = -6$, p is one of 1, -1, 2, -2, 3, -3, 6, -6. So, possible rational zeros are 1, -1, 2, -2, 3, -3, 6, -6.