B02. **MATH 1200: Test #2 (Fall 2011)**

Solution; marking scheme

[6] 1. Consider the sequence $\{c_n\}$ defined by $c_0 = 2$, $c_{n+1} = \frac{c_n}{2}$ for $n \geq 1$.

(a) Show that $\{c_n\}$ is monotonic.

(b) Show that the sequence $\{c_n\}$ is bounded.

(c) Find $\lim_{n\to\infty} c_n$.

Solution.

(a) $c_{n+1} < c_n$ since $\frac{c_n}{2} < c_n$. So the sequence is decreasing.

(b) All members are positive; so the sequence is bounded from below by 0. On the other hand, the sequence is decreasing by (a), so all members are smaller than the first term; so 2 is an upper bound for the sequence.

(c) $c_{n+1} = \frac{c_n}{2}$ gives $\lim_{n\to\infty} c_{n+1} = \lim_{n\to\infty} \frac{c_n}{2}$; so $L = \frac{L}{2}$; so $L = 0$.

[5] 2. (a) State the definition of an oscillating sequence (i.e., what does it mean to say that a sequence $\{c_n\}$ is oscillating).

(b) Write "Yes" if the following sequence is oscillating, write 'No" if it is not oscillating. Do NOT justify.

(i) $2,-2,2,-2,....$

(ii) $\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2},....$

(iii) $\frac{1}{2},-\frac{1}{3},\frac{1}{4},-\frac{1}{5},....$

Solution. (a) In the book.

(b) (i) No, (ii) No, (iii) Yes

[4] 3. You have 500 on your account. You add 100 each month. The annual interest is 12% compounded each month. Write down an explicit formula for your balance after *n*-many months. Do not simplify. [Hint: Recall that if $c_1 = a$ and $c_n = d + rc_{n-1}$, then $c_n = ar^{n-1} + d\left(\frac{r^{n-1}-1}{r-1}\right)$.]

Solution. Note that the monthly interest is 1% or 0.01. In this context we have $c_1 = a = 500$ and $c_n = c_{n-1} + 0.01c_{n-1} + 100$, i.e., $c_n = 1.01c_{n-1} + 100$. Using the formula in the hint,

$$c_n = (1.01)^{n-1}500 + 100\left(\frac{(10.1)^{n-1}-1}{1.01-1}\right).$$