B01. **MATH 1200: Test #1 (Fall 2011)**

Solution; marking scheme

[6] 1. Use mathematical induction to prove the following:

$$4+8+13+\cdots+4n=2n(n+1)\ .$$

Solution.
$n=1$: $4=2(1)(2)$ true.
$n=k$: assume $4+8+13+\cdots+4k=2k(k+1)$.
$n=k+1$: checking if $4+8+13+\cdots+4k+4(k+1)=2(k+1)(k+2)$.
Left hand side: $4+8+13+\cdots+4k+4(k+1)=2k(k+1)+4(k+1)=(k+1)(2k+4)$, and this is obviously equal to $2(k+1)(k+2)$.

[4] 2. Write the following using the sigma notation:

(a) $\sin 1-\sin 2+\sin 3-\sin 4+...+\sin 2011$
(b) $\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\frac{1}{26}+\frac{1}{37}+...$

Solution. (a) $\sum_{k=1}^{2011}(-1)^{k+1}\sin k$

(b) $\sum_{k=1}^{\infty}\frac{1}{k^2+1}$

[5] 3. Write the first 4 terms of the sequence. Find the limit of the sequence or explain why the limit does not exist. Use proper notation for limits.

(a) $\left\{1+\frac{1}{n}\right\}$
(b) $\left\{2+(-1)^n\right\}$

Solution. (a) $2,1+\frac{1}{2},1+\frac{1}{3},1+\frac{1}{4},...;\ \ \lim_{n\to\infty}\left(1+\frac{1}{n}\right)=1$

(b) The sequence is $1,3,1,3,1,3,...$ and it does not approach any fixed number. So, the limit does not exist.