

UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS
MATH/FA 1020 Math in Art
FINAL EXAM; April 16, 2019

LAST NAME: (Print in ink) _____

FIRST NAME: (Print in ink) _____

STUDENT NUMBER: (in ink) _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

A01 Michelle Davidson

A02 Sasho Kalajdzievski

INSTRUCTIONS TO STUDENTS:

Fill in all the information above.

No texts, notes, cell phones or other aids are permitted.

Simple calculators are permitted, as is a ruler and a compass.

Show your work clearly for full marks.

This exam has a title page, 8 pages of questions and 1 blank page at the end for rough work.

Please check that you have all pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 70.

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the reverse side of the page, but clearly indicate that your work is continued there.

Question	Points	Score
1	7	
2	8	
3	7	
4	7	
5	8	
6	9	
7	8	
8	8	
Total:	62	

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EXAMINATION: Math & Art

TIME: 2 HOURS

EXAMINER: Various

Important: The term “construct” in all of the questions means “construct using an unmarked ruler and a compass”. The phrase “unmarked ruler” stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done.

[7] 1. [3] (a) Construct an angle of 45° .

[4] (b) The points A and B lie on one circle, and the points C and D lie on another circle. The two circles are concentric (meaning they have a common centre). Construct their common centre.

A.

C.

B.

D.

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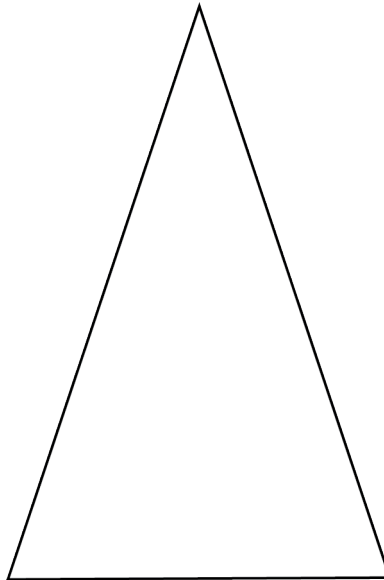
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EXAMINATION: Math & Art

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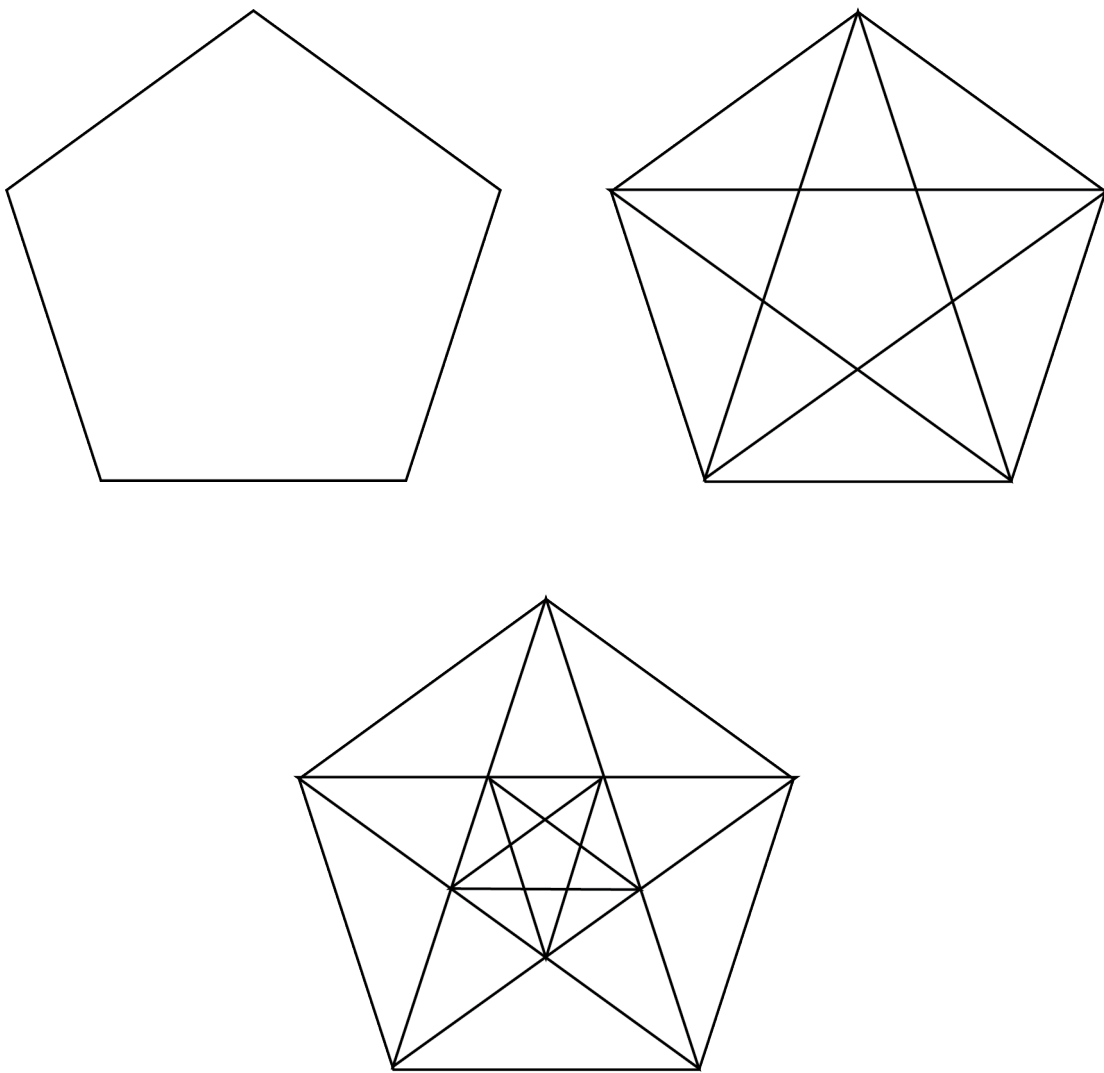
[8] 2. [4] (a) The triangle shown below is an acute golden triangle. Construct a line segment within this triangle such that it subdivides the triangle into two smaller golden triangles.



[4] (b) We are given the 20th and the 23rd Fibonacci numbers: $f_{20} = 6765$ and $f_{23} = 28657$. Find the 21st and the 22nd Fibonacci numbers. Show your work and justify.

[7] 3. [3] (a) The first three steps of the construction of a fractal F are shown in Figures 1, 2 and 3 below. Construct precisely the object we get after performing one more step in the construction of F . You may do your construction on top of Figure 3. Note that the smallest (central) pentagon in Figure 3 is ϕ^4 -times smaller than the largest pentagon in Figure 3 (where ϕ is the golden ratio).

4. (b) The fractal F is obtained by performing infinitely many steps as indicated in part (a) of this question. Show that F is indeed a fractal by identifying the center and the stretching factor of one proper central similarity that sends the fractal F within itself. (Recall that a *proper* similarity is a similarity of stretching factor not equal to 1.)



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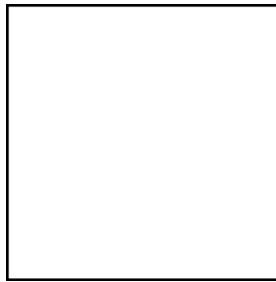
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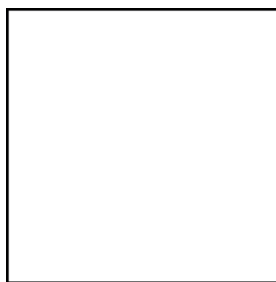
TIME: 2 HOURS

EXAMINER: Various

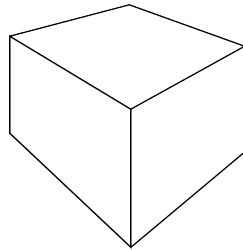
[7] 4. [3] (a) Find the group of symmetries of each of the square shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the lines of reflections. If you use translations, show the vectors of translations, drawing **precisely** at least one of them. [The object is defined by the black points.]



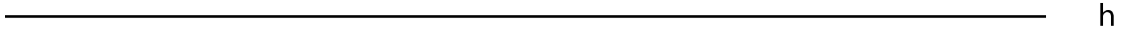
[4] (b) The square from part (a) is shown again below. Extend the square by drawing additional line segments so that the new design has exactly 4 symmetries.



[8] 5. [3] (a) The figure below shows a box on the ground in two-point perspective. Construct the horizon.



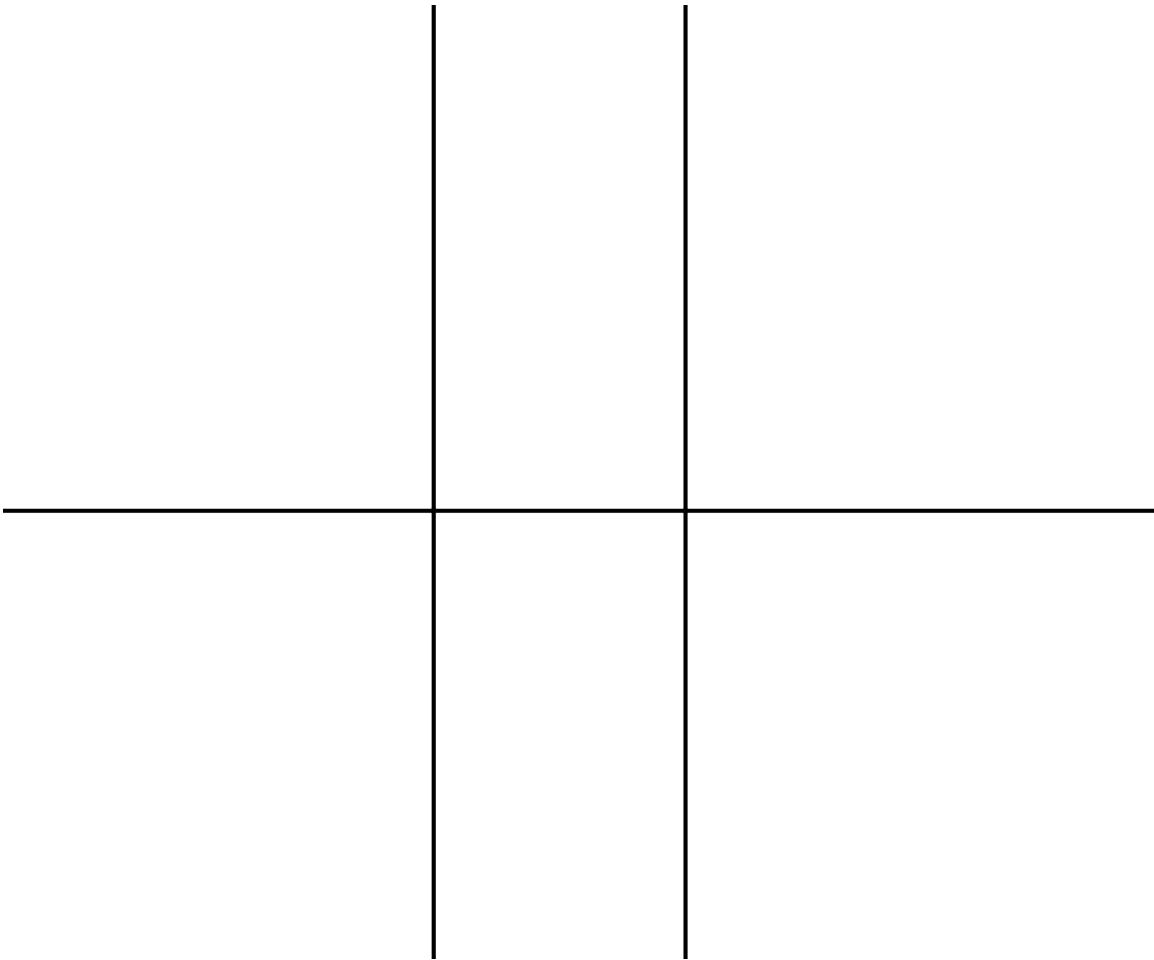
[5] (b) The picture below shows a line AB (on the ground) and the horizon h. Subdivide AB into three equal parts (in perspective).



A.

.B

[9] 6. [6] (a) Construct at least 8 points on a hyperbola. The starting setup for this construction is given below (one horizontal and two vertical lines).



[3] (b) Construct an outline of a parabola using six tangents.

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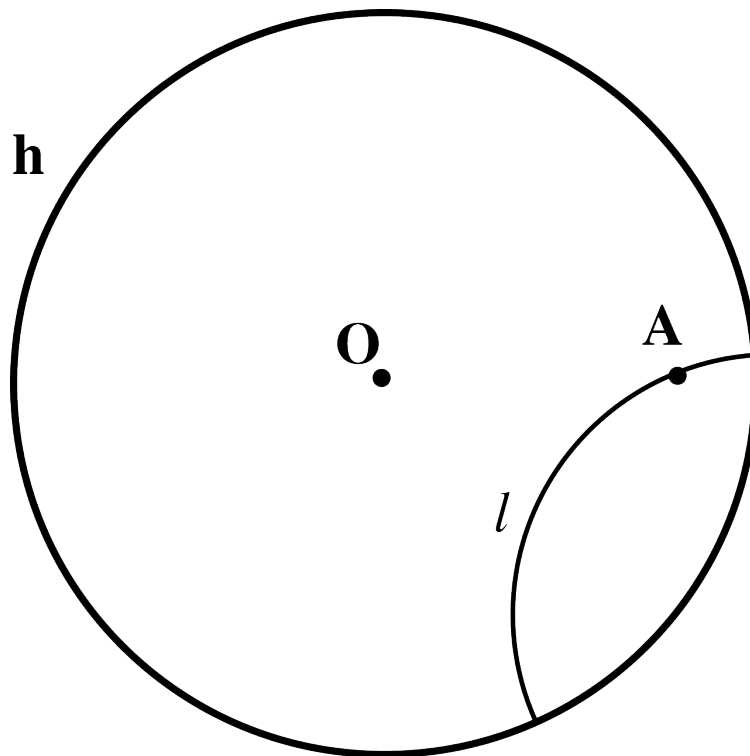
TIME: 2 HOURS

EXAMINER: Various

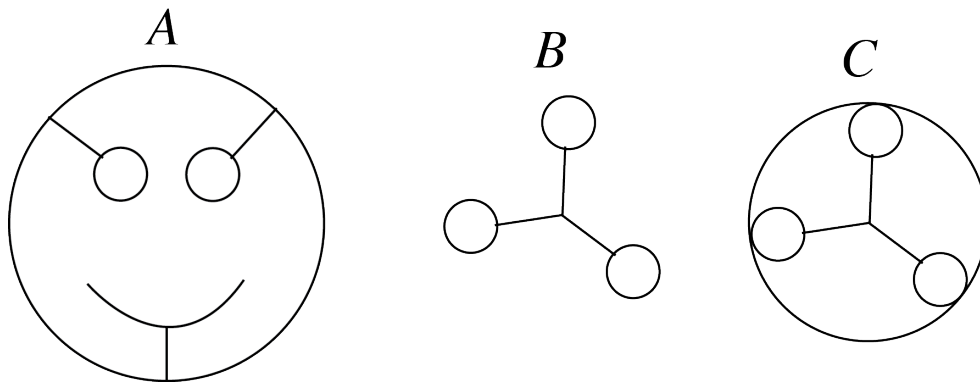
[8] 7. In the Poincare model of hyperbolic geometry shown below h is the horizon and O is the center of h . We are given a hyperbolic line l of the first type, and the point A on l .

[2] (a) Construct two hyperbolic lines that are parallel to l . Label these lines by m , and n

[6] (b) Construct one hyperbolic line passing through A and intersecting the line l at the right angle; label it by s .

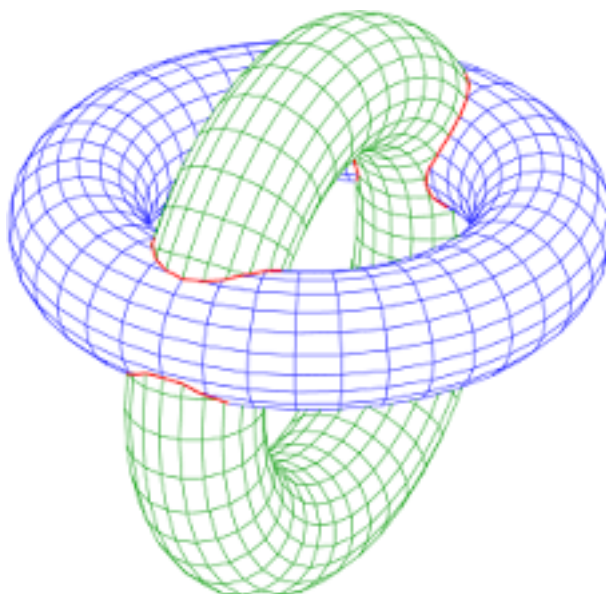


[8] 8. [4] (a) Two of the three designs A , B and C shown below are mutually homotopic. Which two? Justify your answer by drawing at least three in-between sketches showing how one of the objects can be continuously deformed into the other.



[4] (b) Consider the (outer) **surface** of the object shown below (two intersecting tori, one vertical, one horizontal, intersecting in two places).

- i) Find the genus of this surface. Justify by drawing the associated circular cuts.
- ii) Find the Euler characteristic of this surface. Justify by giving the appropriate formula used.



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