UNIVERSITY OF MANITOBA
COURSE: MATH\FA 1020
DATE & TIME: April 16, 2019, 1:30–3:30
DURATION: 2 hour

I agree to follow all test regulations:

Signature (*In Ink*): ______________________

INSTRUCTIONS

I. Calculators are permitted. However, your calculator must NOT have Wi-Fi access, NOT be programmable, and NOT contain pre-programmed functions (i.e. cell phones and graphing calculators are not allowed). If you are unsure if your calculator is allowed, ask your instructor.

II. Student are required to have a compass and an unmarked ruler. For the purposes of this exam, and unmarked ruler is any ruler which is used without reliance on the markings (I.e. a regular ruler is fine, as long as things are constructed and not measured). Students may have other geometry tools.

III. The term “construct” in all of the questions means “construct using an unmarked ruler and a compass”. The phrase “unmarked ruler” stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). **Use words to describe BRIEFLY what you have done.**

IV. NO texts, notes, cellphones, personal music players, WiFi or cell enabled devices, electronic dictionaries or any similar aids/devices are allowed. You may not bring in scrap paper with you.

V. The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 64 points.

VI. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more space, indicate clearly that your work is continued on the back of a page.

VII. Unless instructed otherwise, you must show work (calculations, intermediate steps, etc) to receive full credit.

VIII. **Do not make any marks on the QR codes!**

************************ EXTRA WORK HERE ************************

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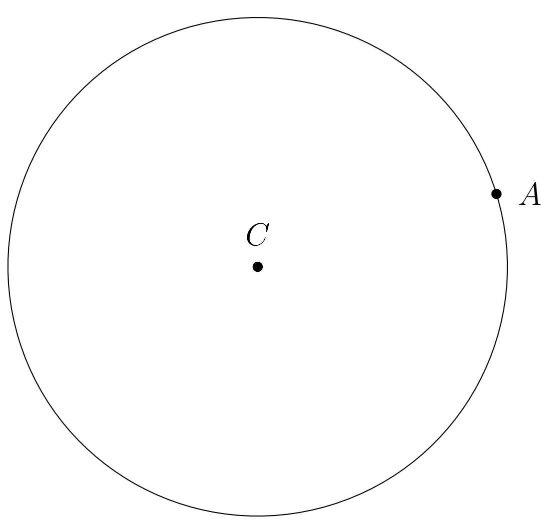
(Instructions repeated for emphasis.) The term "construct" in all of the questions means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). **Use words to describe BRIEFLY what you have done.**

[3] 1. (a) Construct an angle of $15°$.

[3] (b) Construct a circle that passes through the point A and is perpendicular to the given circle (with centre C). Clearly indicate the centre of the circle you have constructed.



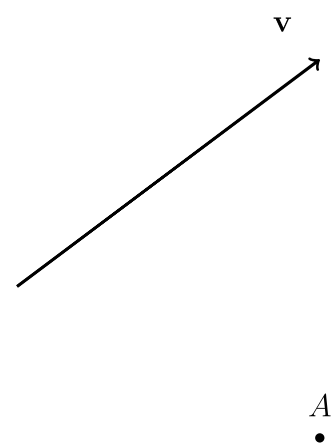
************************ EXTRA WORK HERE ************************

[3] 2. (a) Construct a regular hexagon such that the given line segment below is one of the sides.

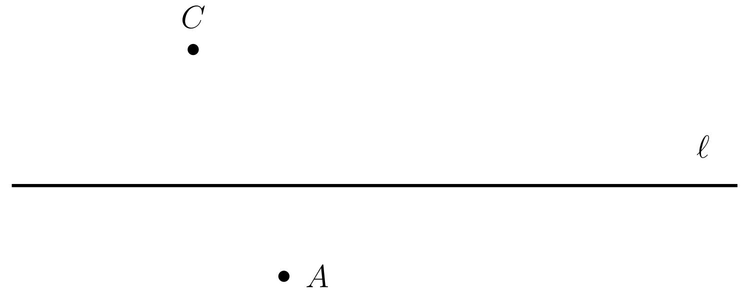
[5] (b) Construct a golden rectangle so that the line segment given below is one of the **shorter** sides.

************************ EXTRA WORK HERE ************************

[2] 3. (a) The vector $\mathbf{v}$ and the point A are shown. Construct the image of A under the translation along the vector $\mathbf{v}$.



[6] (b) The line ℓ and the points A and C are given. We let f be the rotation about C by $60°$, denoted $f = rot(C, 60°)$, and g be the reflection with respect to the line ℓ, denoted $g = ref_\ell$. Construct the image $g(f(A))$ of the point A under the composition of f followed by g.



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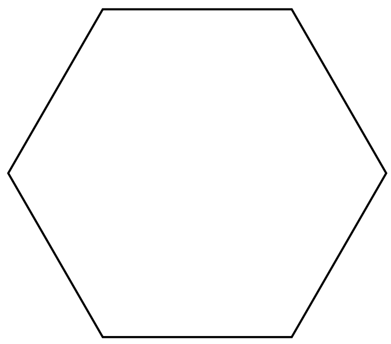
[5] 4. (a) Find the group of symmetries of the hexagon shown below. If you claim a rotational symmetry, indicate the centre of the rotation and the angle of rotation (in degrees). If there are reflections, show (and label) the lines of reflections. If you use translations, show the vectors of translations, drawing **precisely** at least one of them.



[3] (b) Sketch any object in the plane that has exactly 5 symmetries. Write the group of 5 symmetries of the object that has been sketched.

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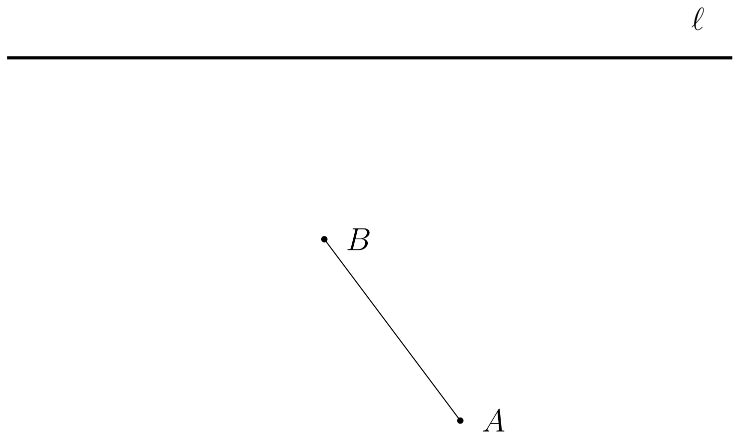
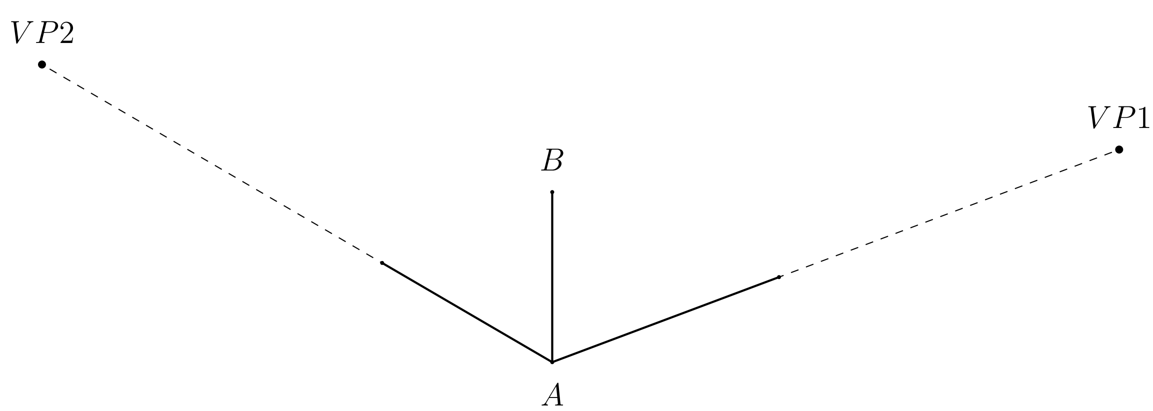
[3] 5. (a) We are given the horizon h and the line segment AB (in perspective). Subdivide AB into three equal parts (in perspective).

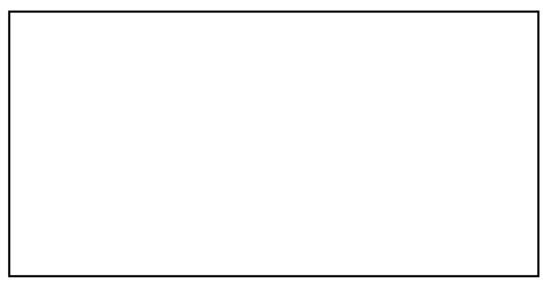


[5] (b) Three adjacent vertices of a box are shown in 2-point perspective, with vanishing points $VP1$ and $V2$ (the edge AB is vertical and parallel to the drawing plane). Construct the remaining edges of the box. (Note: The dashed lines are not part of the box.)

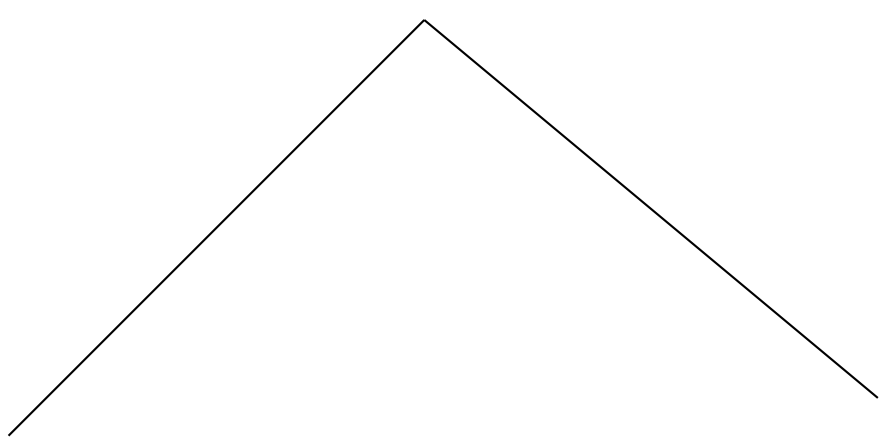


************************ EXTRA WORK HERE ************************

[5] 6. (a) Construct at least 4 (tangent) lines outlining the contour of the top of the ellipse inscribed in the given rectangle. (**No** need to construct the bottom part of the ellipse, so that the constructed half of an ellipse should be in the top half of the given rectangle.)



[3] (b) Subdivide each of the two segments shown below into 4 equal parts, then use these points to construct an outline of a parabola by drawing precisely 4 tangential line segments.

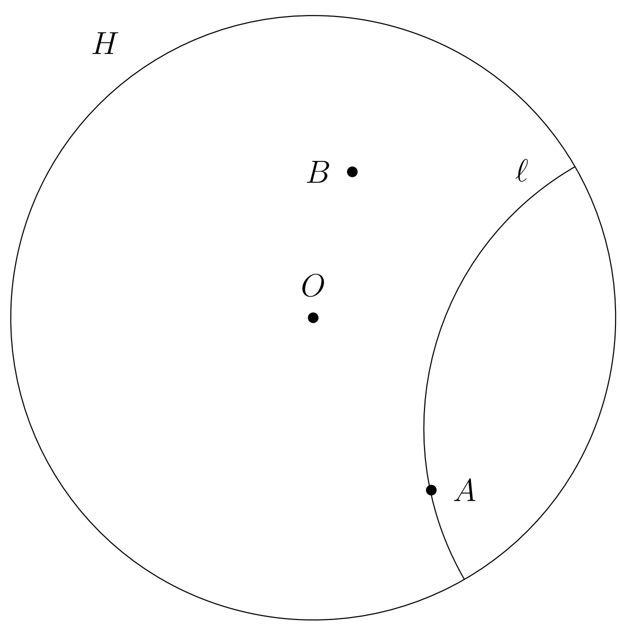


************************ EXTRA WORK HERE ************************

7. We are given a hyperbolic line ℓ, the point A on that hyperbolic line, and a point B outside the line ℓ.

[2] (a) Construct one hyperbolic line parallel to ℓ and passing through B. Label that line by m.

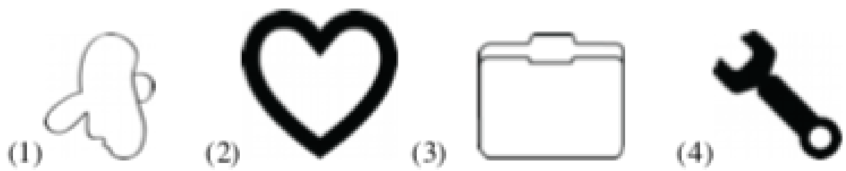
[7] (b) Construct the hyperbolic line passing through A and perpendicular to ℓ. Label it by n.



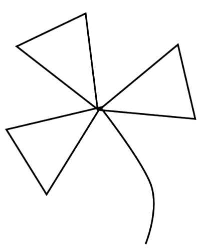
************************ EXTRA WORK HERE ************************

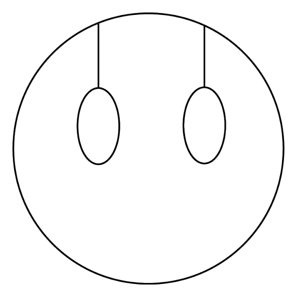
8. The objects depicted in parts (a) and (b) of this question consist of the black - coloured points only.

[2] (a) Which of the following four designs are mutually homotopic? (No justification required.)



[3] (b) Show that the two designs shown below are homotopic by drawing at least three in-between sketches showing the one of the objects can be continuously deformed into the other.

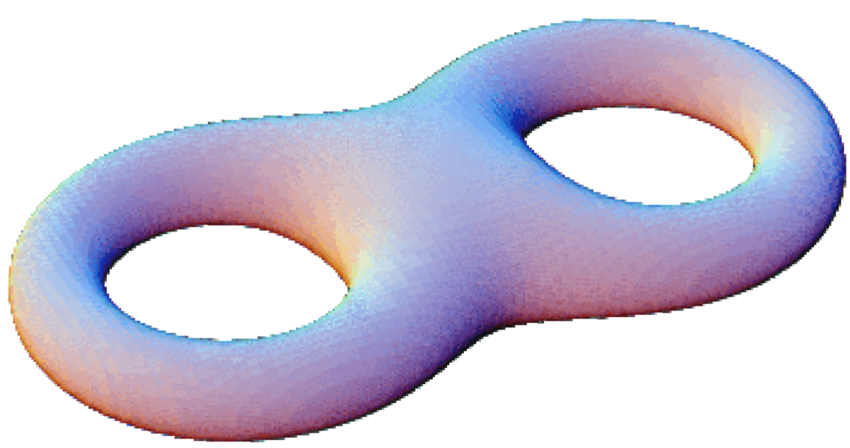




[4] (c) Consider the surface of the double-donut (connected sum of two tori).

i. Find the genus of this surface. Justify by drawing the associated circular cuts.

ii. Find the Euler characteristic of this surface. Justify by giving the appropriate formula used.



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