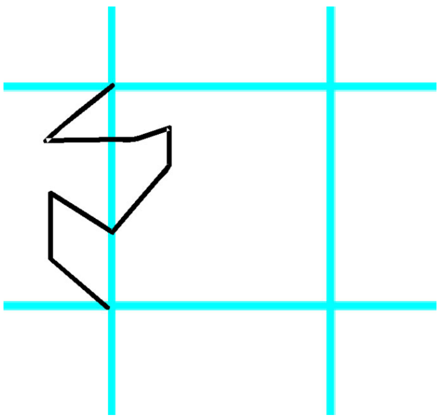
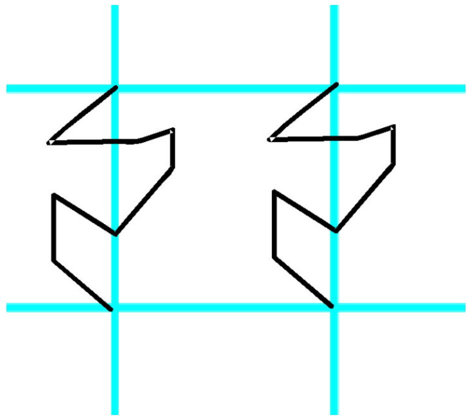
Recipes for Escher-type tilings.

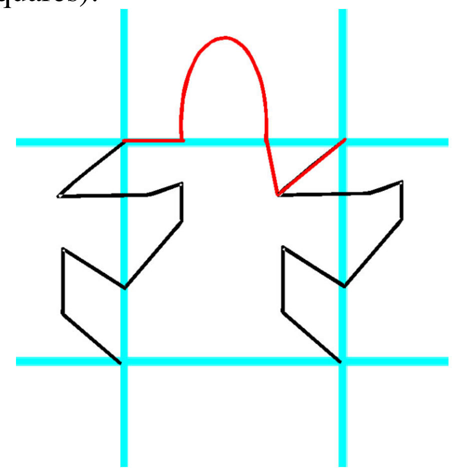
Recipe #1.
Start with a square (we keep an eye on the underlying regular tiling by squares).



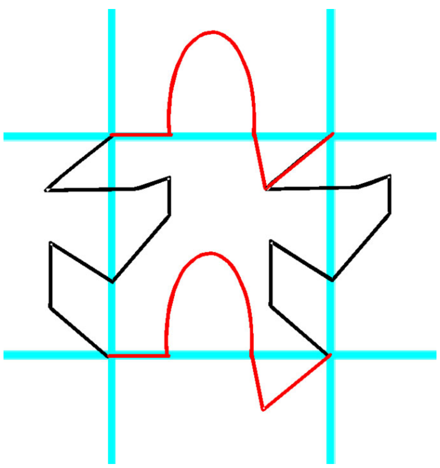
1. Start with any curve joining the two vertices on the left hand side of the



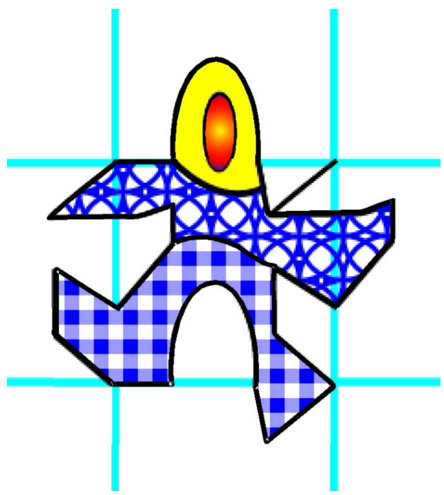
2. Translate the broken line as shown.



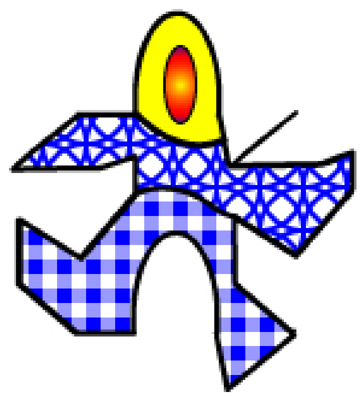
3. Now join the two top vertices of the square with any other curve.



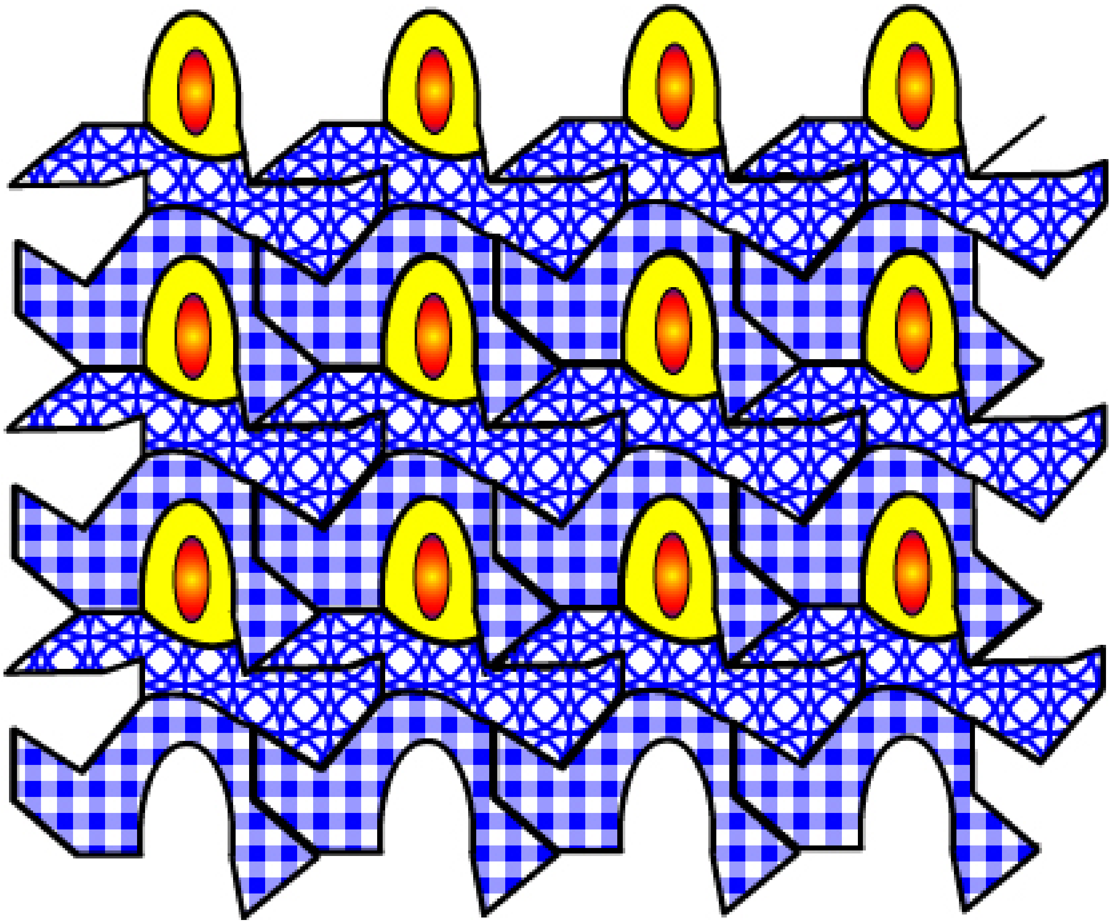
4. Translate the last broken line downwards as shown .



5. Draw or colour the interior of the contour any way you want.



6. We do not need the background square grid any more.



7. Our construction guarantees that there is a tiling of the plane with our shape as the only tile.

Take a look at steps 2 and 4 above: in step 2 we connect two lower vertices of the square in such a way that the broken line we use is symmetric to the broken line used in the first step, and, equally importantly, the part of the broken line in step 2 ***within*** the square is similar to the part of the broken line in step 1 that is ***out*** of the square. Same is true for the broken lines in steps 3 and 4. As long as we have pairs of lines that fulfill these two properties the tile we make can be used to tile the plane.

In the last example the underlying regular tiling was made of one type of a tile – a square. The recipe will work for the other two regular monochromatic tilings (triangles and hexagons) – as long as the analogue of the above mentioned properties is satisfied. We call these properties ***a recipe for Escherisms*** and identify them explicitly[1]:

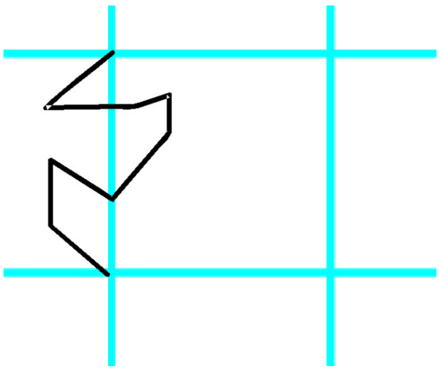
1. The contour of the tile we construct can be subdivide into pairs of symmetric curves.

2. In every pair of similar curves identified in 1, the part of the curve *within* the underlying square (equilateral triangle, hexagon) is similar to the part of the other curve of the pair that is *out* of the square (triangle, hexagon respectively).

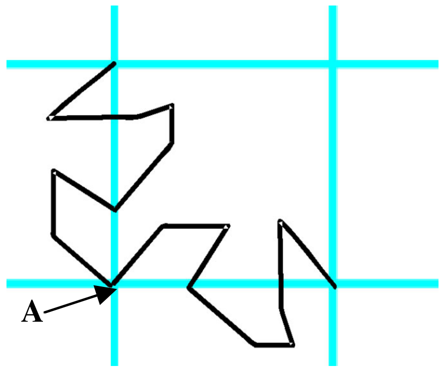
[1] The recipe we give is incomplete. Mathematically inclined may want to try to find an example of a tile cooked out of a hexagon and according the recipe , yet not allowing a tiling of the plane.

Recipe #2.

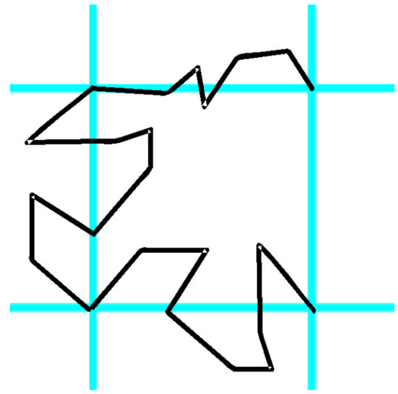
We will now modify the first example to identify the freedom we have under our recipes for Escherisms.



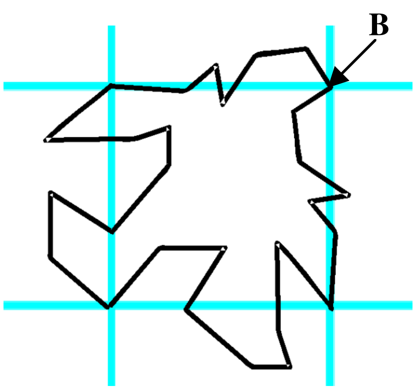
1. Any curve joining the left-hand side vertices.



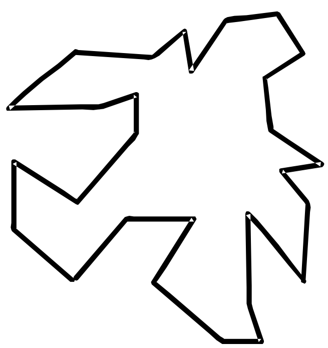
2. This time we rotate the curve around the point A and by –90º (negative sign since we go clockwise). Notice that the two curves satisfy both properties in the recipe for Escherisms.



3. Any other curve joining the two top vertices of the square.

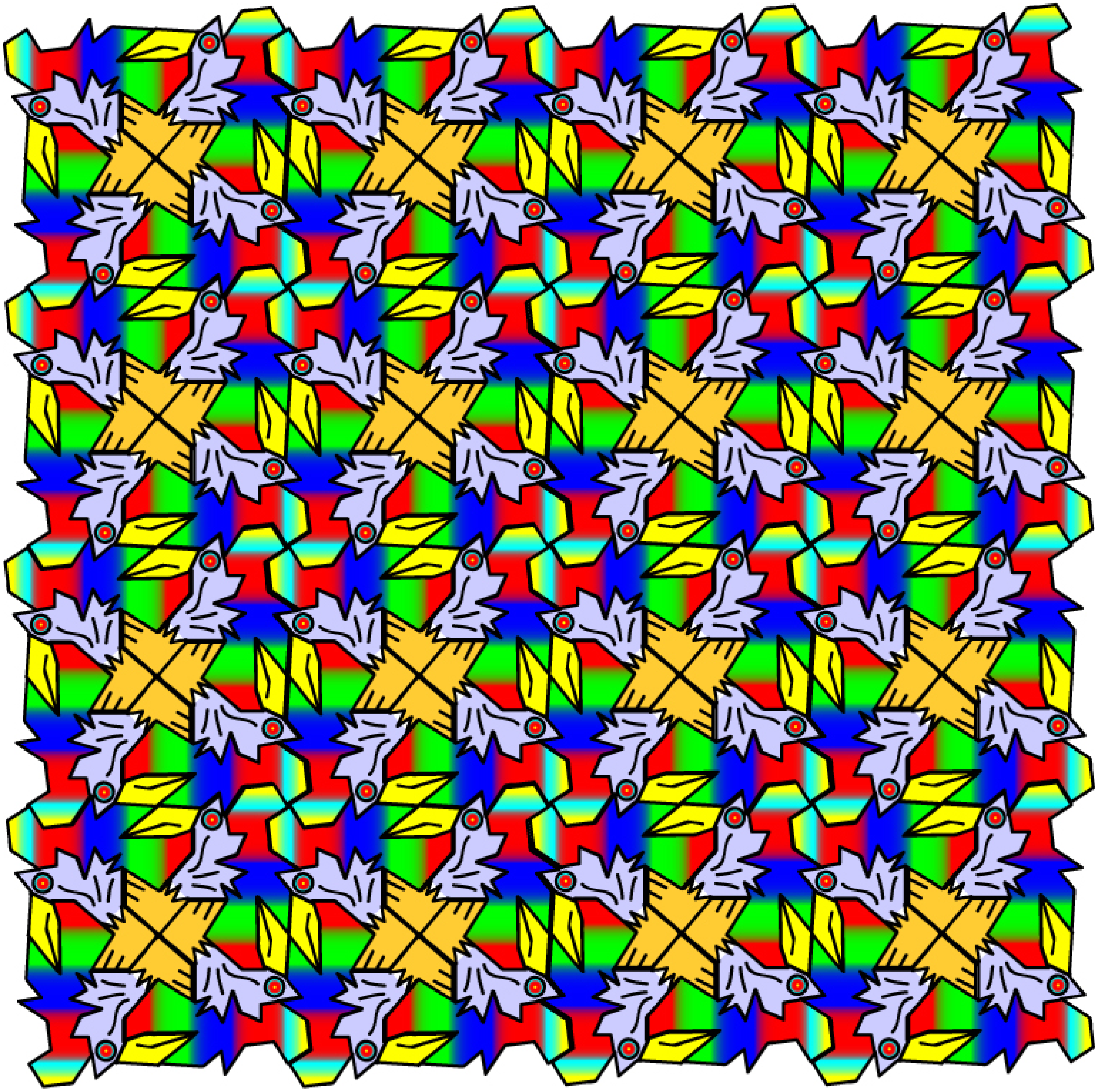


4. Rotate around B by 90º. We get another curve and the last two curves satisfy the two properties in the recipe.



5. Erase the background

6. Draw or colour whatever you want inside the our tile.



7. Rotate translate copies of our tile until they nicely fit – our construction guarantees they will match and cover all of the plane