Western Canada Linear Algebra Meeting
Programme

University of Manitoba

May 14-15, 2016

Organising Committee

Shaun Fallat, Doug Farenick, Hadi Kharaghani, Steve Kirkland, Peter Lancaster, Michael Tsatsomeros, Pauline van den Driessche

Local Organisers:
Jane Breen, Sarah Plosker

Funding

The organisers gratefully acknowledge the sponsors of WCLAM 2016: ILAS, the Pacific Institute for the Mathematical Sciences, and the University of Manitoba.

Invited Speakers

Rajesh Pereira (University of Guelph),
Rachel Quinlan (National University of Ireland Galway), and
Kevin Vander Meulen (Redeemer University College, ILAS Lecturer).

Location

All lectures will be held in the Robert Schultz Lecture Theatre in St. John's College, room 142

1 Schedule

Saturday, May 14, 2016

8:15-9:00	**Breakfast (provided) and Registration**
	Chair: Steve Kirkland
9:00-10:00	**Kevin N. Vander Meulen (invited)**, Spectral Analysis of Matrix Patterns
10:00-10:30	**Coffee**
	Chair: Shaun Fallat
10:30-11:00	**Kris Vasudevan**, Signed graph Laplacians: From small-world to hierarchical modular graphs
11:00-11:30	**John Sinkovic**, The inertia bound is not always tight
11:30-12:00	**Colin Garnett**, Combinatorial conditions that preclude SAPpiness
12:00-12:30	**Xavier Martinez-Rivera**, Classification of families of pr- and epr-sequences
12:30-1:30	**Lunch (provided)**
	Chair: Pauline van den Driessche
1:30-2:30	**Rachel Quinlan (invited)** Adventures in Nilpotent Matrix Spaces
2:30-3:00	**Samir Raouafi** Pseudospectra and the Behavior of Non-normal Matrices
3:00-3:30	**Douglas Farenick** A multivariable analogue of Ando's theorem on the numerical radius
3:30-4:00	**Coffee**
	Chair: Michael Tsatsomeros
4:00-4:30	**M. Rajesh Kannan** Some properties of semipositive matrices
4:30-5:00	**Steve Butler** Some comments on cospectral graphs for the normalized Laplacian
5:00-7:00	**Supper (provided)**

Sunday, May 15, 2016

8:15-9:00	**Breakfast (provided)**
	Chair: Doug Farenick
9:00-10:00	**Rajesh Pereira (invited)**, Doubly Stochastic Matrices and Majorization
10:00-10:30	**Coffee**

Chair: Sarah Plosker

10:30-11:00	**Nathaniel Johnston**, Some linear algebra questions arising from quantum coherence
11:00-11:30	**Xiaohong Zhang**, Bounds on fidelity of state transfer with respect to errors
11:30-12:00	**Peter Loly**, Matrix norms of angular momentum matrices in quantum mechanics
12:00-1:00	**Lunch (provided)**

Chair: Shaun Fallat

1:00-1:30	**Keivan Hassani Monfared**, Some Inverse Eigenvalue Problems for Graphs (IEPG)
1:30-2:00	**Guanglong Yu**, Some extremal values on graph Q-spectra
2:00-2:30	**Jephian C.-H. Lin** Using a new zero forcing process to guarantee the Strong Arnold Property
2:30-3:00	**Louis Deaett**, Minimum rank for circulant graphs and matrices
3:00-3:01	**Closing remarks**

2 Abstracts (alphabetical by speaker)

Some comments on cospectral graphs for the normalized Laplacian

Steve Butler

In spectral graph theory the goal is to understand as much as we can about a graph given the eigenvalues of some associated matrix. There are many different such matrices and among them the normalized Laplacian matrix (or equivalently the probability transition matrix) has some of the most unexpected properties. This includes (but is not limited to) the existence of cospectral graphs with differing number of edges, including graphs cospectral with subgraphs, as well as graphs which are cospectral with regular graphs. We will highlight some very simple tools of scaling and twins to help establish such pairs of graphs.

Minimum rank for circulant graphs and matrices

Louis Deaett, Seth Meyer

The *minimum rank* of a simple graph G is the smallest rank of a symmetric matrix with support equal to that of the adjacency matrix of G. When G is a circulant graph, we seek the smallest rank of such a matrix that is additionally a circulant matrix. For certain families of circulants, the more general minimum can always be achieved under this additional restriction.

We also consider the further restriction to matrices that are not merely symmetric, but positive semidefinite. A positive semidefinite matrix associated with G corresponds to an orthogonal representation of G, an assignment of a vector to each vertex such that non-orthogonality occurs precisely for adjacent pairs. The matrix is a circulant matrix when the representation has a sort of generalized rotational symmetry. A representation with this symmetry is equivalent to a polynomial with nonnegative coefficients whose zeros on the unit circle are determined by the graph. This connection with polynomials serves as a powerful tool. For example, it allows us to compute the minimum for every circulant whose number of vertices is prime.

A multivariable analogue of Ando's theorem on the numerical radius

Douglas Farenick, Ali Kavruk, Vern Paulsen

A classic theorem of T. Ando characterises operators that have numerical radius at most 1 as operators that admit a certain positive 2×2 operator matrix completion. In this lecture, I will introduce the notion of numerical radius for n-tuples $X = (X_1, \ldots, X_n)$ and show that Ando's theorem may be formulated (via a positive completion problem for certain $n \times n$ operator matrices) and proved in this multivariable setting. Some unexpected connections in operator algebra theory arise for both the proof and the application of this extended Ando theorem.

Combinatorial conditions that preclude SAPpiness

Louis Deaett, Colin Garnett

It is well known that a complex zero-nonzero pattern cannot be spectrally arbitrary if its digraph doesnt have at least two loops and at least one two cycle, or at least three loops. This talk focuses on several other combinatorial conditions on the digraph that preclude it from being spectrally arbitrary. In particular we are sometimes able to reduce the number of unknown entries to be below the threshold of $2n-1$. We are also investigating structures that allow us to conclude that a monomial multiple of one of the coefficients in the characteristic polynomial is in the ring generated by the other coefficients.

Some Inverse Eigenvalue Problems for Graphs (IEPG)

Keivan Hassani Monfared, Ehssan Khanmohammadi, Sudipta Mallik, Bryan L. Shader

Inverse eigenvalue problems for graphs ask about the existence of a matrix in a family $\mathcal{F}$ with a given graph G and some spectral property $\mathcal{P}$. In this talk we show if there is a generic solution for the problem when a subgraph H of G has a much simpler structure than G, then one can use the Jacobian method to show the problem with any supergraph of H, in particular G, will have a solution. Here, the family $\mathcal{F}$ is the set of real (not necessarily symmetric) matrices with a given graph G on finitely many vertices. We will focus on the case that the $\mathcal{P}$ is the property that A has a given spectrum $\Lambda \subset \mathbb{C}$.

Some linear algebra questions arising from quantum coherence

Jianxin Chen, Nathaniel Johnston, Chi-Kwong Li, Sarah Plosker

There are two key ingredients that make quantum information theory different from classical information theory: entanglement and superpositions. Various measures of quantum entanglement have been investigated for years, but measures of coherence (i.e., "how superpositioned" a quantum state is) are a bit less well-studied. In this talk we will discuss, from a linear algebra point of view, three measures of quantum coherence that have been proposed, and we will answer some open questions about them.

Some properties of semipositive matrices.

Projesh Nath Choudhury, M. Rajesh Kannan, K.C. Sivakumar

A real matrix A is called *semipositive*, if there exists a vector $x \geq 0$ such that $Ax > 0$ (entry wise), and is called *minimally semipositive* if it is semipositive and no column-deleted submatrix of A is semipositive. In this talk, we shall discuss some properties of semipositive and minimal semipositive matrices in connection with interval hull of matrices, generalized inverses and generalized principal pivot transforms.

Using a new zero forcing process to guarantee the Strong Arnold Property

Jephian C.-H. Lin

A given symmetric matrix A is said to have the *Strong Arnold Property* (SAP) if the zero matrix is the only symmetric matrix Xthat satisfies $A \circ X = O$, $I \circ X = O$, and $AX = O$. The SAP plays a key role in ensuring the minor-monotonicity of the Colin de Verdière type parameters μ, ν, and ξ. To understand the SAP, the connection between the zero-nonzero pattern of a symmetric matrix and the adjacency of a simple graph provides important information. In this talk, a method of using the graph structure to guarantee the SAP will be introduced, with the help of the zero forcing process.

Matrix norms of angular momentum matrices in quantum mechanics

Peter Loly

The eigenvalue and singular value spectra, including sums of their powers (matrix norms), of the quantum angular momentum matrices are studied. What is surprising here is that even though the angular momentum matrix elements are not generally integer, since they contain square roots and/or imaginary elements, we have been able to find integer matrix norms for all spin quanta (1/2,1,3/2,2,5/2,...). Their characteristic polynomials have integer coefficients when the "Pauli scale" is used. When these are looked up in N.J.Sloane's OEIS (Online Encyclopedia of Integer Sequences) they point back to early work on nuclear magic numbers!

Classification of families of pr- and epr-sequences

Xavier Martinez-Rivera

The *principal minor assignment problem* asks the following question: can we find an $n \times n$ real symmetric matrix having prescribed principal minors. An attempt to simplify this problem led to the introduction of two sequences for a symmetric (or complex Hermitian) matrix. The *principal rank characteristic sequence* of an $n \times n$ symmetric matrix B is $r_0]r_1 \cdots r_n$, where, for $k = 1, 2, \ldots, n$, $r_k \in \{0, 1\}$ and $r_k = 1$ if and only if B has a nonzero principal minor of order k, while $r_0 = 1$ if and only if B has a 0 on its main diagonal (otherwise $r_0 = 0$). The *enhanced principal rank characteristic sequence* of an $n \times n$ symmetric matrix B is $\ell_1 \ell_2 \cdots \ell_n$, where ℓ_k is $\mathtt{A}$ (respectively, $\mathtt{N}$) if all (respectively, none) the principal minors of order k are nonzero; if some but not all are nonzero, then $\ell_k = \mathtt{S}$. Results regarding the attainability of certain classes of sequences are discussed, as well as restrictions for some subsequences to appear in an attainable sequence.

Doubly Stochastic Matrices and Majorization

Rajesh Pereira

We discuss the recent solution of the $n = 4$ case of the Perfect-Mirsky conjecture on the eigenvalues of doubly stochastic matrices and explain its connection to both the majorization order and to group representation theory. We use the connection between doubly stochastic matrices and majorization to define the spectrum of different majorization relations (multivariate majorization, directional majorization etc...) which we explore. Several open questions will also be discussed.

Adventures in Nilpotent Matrix Spaces

Rachel Quinlan

It is 58 years now since Gerstenhaber initiated the detailed study of linear spaces of nilpotent matrices, with a series of four articles investigating dimension bounds and extremal examples, subject in some cases to additional conditions. Since then the subject has seen considerable development and it continues to yield some interesting surprises. This talk will include a (selective and non-exhaustive) survey of some old and newer discoveries in the area, with particular emphasis on finite fields and on the oddity of the even prime 2.

Pseudospectra and the Behavior of Non-normal Matrices

Samir Raouafi

Pseudospectra are a useful tool for studying matrices and linear operators. The traditional tool is the spectrum. It reveals information on the behavior of normal matrices and operators. However, it is less informative in the non-normal case. Pseudospectra have nevertheless proved to be a powerful tool to investigate non-normal matrices and operators. This talk will start with a brief introduction to the theory of pseudospectra. We will then discuss the behavior of non-normal matrices using pseudospectra.

The inertia bound is not always tight

John Sinkovic

The Cvetković bound gives an upper bound on the independence number of a graph using the number of positive and negative eigenvalues of the adjacency matrix. In fact any weighted adjacency matrix may be used in the formula. When is the bound tight? There are many examples which are known, including all perfect graphs. Elzinga and Gregory (2010) have shown that all graphs on 10 or fewer vertices as well as vertex transitive graphs up to 12 vertices have a weighted matrix which makes the bound tight. I will explain why the bound cannot be tight for the Paley graph on 17 vertices.

Spectral Analysis of Matrix Patterns

Kevin N. Vander Meulen

In 2000, the concept of spectrally arbitrary sign pattern was introduced. I plan to highlight some of the techniques and results that have been developed over the last 16 years. The survey will include variations that focus on the non-signed combinatorial structure as well as variations that focus on inertia. Open problems will be available.

Signed graph Laplacians: From small-world to hierarchical modular graphs

Michael Cavers, Kris Vasudevan

Small-world graphs and hierarchical modular networks have drawn considerable interest in recent years in the areas of mathematics, physics, sociology and biology. Here, we introduce their signed graph Laplacians. We study how the differing ratios of negative interactions to positive interactions affect the spectral properties of the Laplacians and the structural characteristics of the networks. Furthermore, we investigate the influence of model parameters on the Kuramoto model dynamics on the signed graphs.

Some extremal values on graph Q-spectra

Guanglong Yu

Recently, the problems about the extreme theory on graph spectrum have been investigated extensively. This talk will start with a simple introduction about the extreme theory of graph Q-spectrum. Most of the talk, however, will be dedicated to very recent results on Q-spectral radius and the least Q-eigenvalue

Bounds on fidelity of state transfer with respect to errors

Whitney Gordon, Steve Kirkland, Chi-Kwong Li, and Sarah Plosker, Xiaohong Zhang

Quantum state transfer has been proposed as a "data bus" for quantum computation. A state is prepared and placed on one spin of a spin network and read out later on another spin in the network. The fidelity, which is a measurement of the closeness between two quantum states, is used to determine the accuracy of the state transfer. Ideally the fidelity is 1, representing perfect state transfer. But in reality, there will be errors in both the readout time and the edge weights. We want to bound this effect so that small perturbations of readout time and edge weights do not drastically reduce the fidelity. In this talk, we give some bounds on the fidelity of state transfer in the above two cases, respectively. At the heart of the analysis are the numerical range of a matrix and matrix norms.

3 Participants

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