Rectangular powers and Ramsey theory

Bob Quackenbush

ABSTRACT

For a finite structure $A$ (e.g., a graph, poset, group, or lattice), let its set of finite powers be $\text{Pow}(A) = \{ A^n \mid n \geq 0 \}$ with $P_{m,n}(A)$ the set of all substructures of $A^n$ isomorphic to $A^m$.

Choose positive integers $n, m, k, c$ with $n > m > k$. Then we call an onto map $\Delta : P_{k,n}(A) \to [c] = \{1, \ldots, c\}$ a $c$-colouring. We seek $B \in P_{m,n}(A)$ such that the restriction of $\Delta$ to $\{ C \in P_{k,m}(A) \mid C \subseteq B \}$ is constant; such a $B$ is said to be monochromatic with respect to $\Delta$.

I will discuss the positive and negative results of this quest, couched in the language of rectangular powers and polymorphism clones.