Truth Tables

A truth table can be used to show whether a statement is true. To check the validity of an argument using a truth table, use the following four steps:

1. Symbolize each statement of the argument with a letter (usually p, q, r, $\ldots$).
2. Symbolize the premises and conclusion using logical connectives (e.g., $p \wedge (q \vee \neg r)$, etc.).
3. Construct the truth table to have a column for each premise and the conclusion.
4. If the truth table shows that the conclusion is true whenever all premises are true, then the argument is valid. If not, the argument is invalid.

Example. *Use a truth table to determine the validity of the argument "If it is the weekend, and it is not raining, then Jason will play golf".*

Solution:
Following the above steps:

1. p : It is the weekend.
   q : It is raining.
   r : Jason will play golf.
2. The premise of this statement can be symbolized by $p \wedge (\neg q)$. The conclusion of the statement is r.
3.

				(premise)	(conclusion)
p	q	r	$\neg q$	$p \wedge (\neg q)$	r
0	0	0	1	0	0
0	0	1	1	0	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	0	0	1

4. From our truth table, we conclude that the argument is not valid, because in line 5 of the table, we have a case where the premise is true, but the conclusion is false.

Truth tables can also be used to determine if a statement is a tautology or a contradiction (it could also be neither).

Example. *Use a truth table to determine if the statement* $\neg(p \vee q) \longrightarrow (\neg p \vee \neg q)$ *is a tautology.*

Solution:

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(p \vee q) \longrightarrow (\neg p \vee \neg q)$
0	0	0	1	1	1	1	1
0	1	1	0	1	0	1	1
1	0	1	0	0	1	1	1
1	1	1	0	0	0	0	1

Since the final column, representing $\neg(p \vee q) \longrightarrow (\neg p \vee \neg q)$ is has a 1 in every row, the statement is a tautology.

Finally, truth tables are also used to prove that two statements are logically equivalent. In this case, the columns representing the two statements should be identical.

Example. *Using a truth table, prove that* $[p \wedge (q \vee r)] \iff [(p \wedge q) \vee (p \wedge r)]$

Solution:

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Since the columns representing $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$, we have shown that the statements are logically equivalent.

Exercises

1. Use a truth table to determine the validity of the argument "If it is sunny outside and Kevin does not need to Superstore or to Polo Park, then Kevin will meet Alice at 2pm."
2. Use a truth table to determine if $p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$ is a tautology.
3. Using a truth table, prove that $[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$.

Come to the MHC for help with any of these problems!