Math Help Centre

Related Rates Problems

1. Each side of a square is increasing at a rate of 6cm/sec. At what rate is the area of the square increasing when the area is 16cm^2?

2. A particle is moving along a hyperbola $xy = 8$. As it reaches the point $(4, 2)$, the y-coordinate is decreasing at a rate of 3cm/sec. How fast is the x-coordinate changing at that instant?

3. If a snowball melts so that its surface area decreases at a rate of 1cm^2/min, find the rate at which the radius decreases when the diameter is 10cm.
Hint: The formula for the surface area of a sphere is $A = 4\pi r^2$.

4. A Ferris wheel with a radius of 10m is rotating at a rate of one revolution every 2 minutes. How fast is a rider rising when his seat is 16m above ground level?
Note: For this question, assume you are 0m above the ground when you are at the lowest point of the Ferris wheel.

5. Two sides of a triangle have lengths 12m and 15m. The angle between them is increasing at a rate of $2°$/min. How fast is the length of the third side increasing when the angle between the sides of fixed length is $60°$?
Hint: Recall the cosine rule:
$$a^2 = b^2 + c^2 - 2bc\cos A$$

6. Air is being pumped into a spherical balloon at a rate of 5cm^3/sec. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20cm.
Hint: The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

7. A block of ice maintains the shape of a cube as it melts, resulting in its volume decreasing at a rate of 10cm^3 per minute. At what rate is the surface area changing when the block has dimensions 10cm $\times$ 10cm $\times 10$cm?

8. Sand is poured into a conical pile at a rate of 20m^3 per minute. The diameter of the cone is always equal to its height. How fast is the height of the conical pile increasing when the pile is 10m high?
The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

9. An airplane is flying towards a radar station at a constant height of 6km above the ground. If the distance s between the airplane and the radar station is decreasing at a rate of 400km per hour when $s = 10$km, what is the horizontal speed of the airplane?

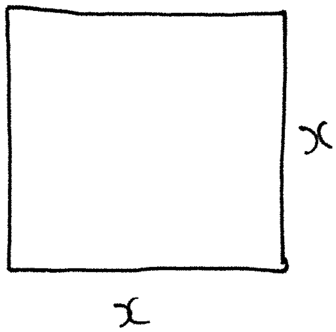
10. Turtle A is walking west, along a straight road, at a speed of 20m/hour and turtle B is walking north along a straight road at 30m/hour. Both are headed for the intersection of their paths. At what rate is the distance between the turtles changing when turtle A is 4m and turtle B is 3m from the intersection of the two roads?

Answers

1. 48 cm^2/sec.
2. 6 cm/sec.
3. $\dfrac{1}{40\pi}$ cm/min.
4. 8π m/min.
5. $\dfrac{60}{\sqrt{7}}$m/min.
6. $\dfrac{1}{80\pi}$cm/sec.
7. -4cm^2/ sec.
8. $\dfrac{4}{5\pi}$m/min.
9. 500km/hour.
10. -34m/hour.

Related Rates Problems

1.



we're given $\frac{dx}{dt} = 6$.

we want $\left.\frac{dA}{dt}\right|_{A=16}$.

Formula for area of square : $A = x^2$.

$$\frac{dA}{dt} = 2x\frac{dx}{dt}.$$

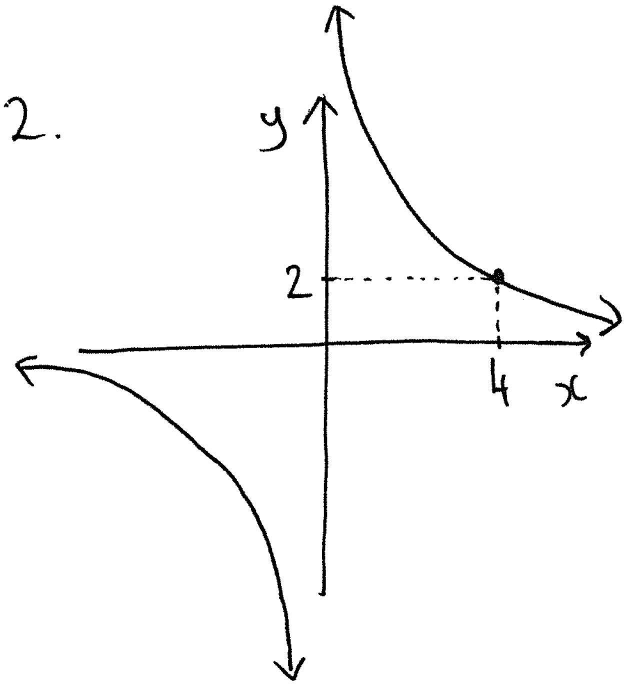
when $A = 16$, $x = 4$.

$$\Rightarrow \left.\frac{dA}{dt}\right|_{A=16} = \left.\frac{dA}{dt}\right|_{x=4}$$

$$\left.\frac{dA}{dt}\right|_{x=4} = 2(4)(6) = 48\,cm^2/sec.$$

The area of the square is increasing at a rate of $48\,cm^2/sec$ when the area is $16\,cm^2$.

2.



We're given $\left.\frac{dy}{dt}\right|_{(4,2)} = -3$

We want $\left.\frac{dx}{dt}\right|_{(4,2)}$

Equation relating x and y: $xy = 8$

$$\Rightarrow \quad y = \frac{8}{x}.$$

$$y = 8x^{-1} \quad \Rightarrow \quad \frac{dy}{dt} = -8x^{-2}\frac{dx}{dt}$$

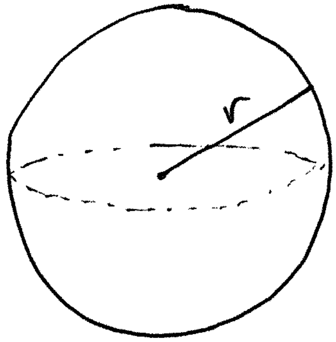
$$\Rightarrow \quad \frac{dy}{dt} = \frac{-8}{x^2}\frac{dx}{dt}$$

$$\Rightarrow \quad \frac{dx}{dt} = -\frac{x^2}{8}\frac{dy}{dt}.$$

$$\Rightarrow \quad \left.\frac{dx}{dt}\right|_{(4,2)} = -\frac{(4)^2}{8}(-3) = 6 \text{ cm/sec}$$

The x-coordinate is changing at a rate of 6cm/sec at the point (4, 2).

3.



we're given $\frac{dA}{dt} = -1$

we want $\left.\frac{dr}{dt}\right|_{r=5}$

Diameter 10 $\Rightarrow$ Radius 5.

$A = 4\pi r^2 \Rightarrow \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

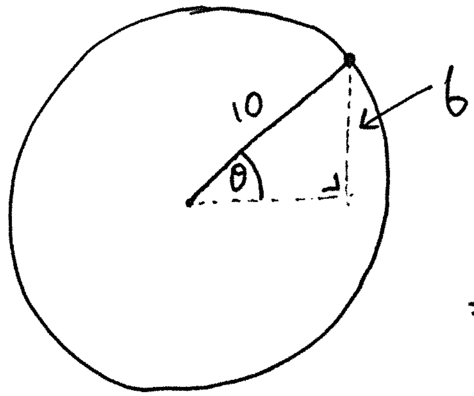
$\Rightarrow \frac{dr}{dt} = \frac{1}{8\pi r} \cdot \frac{dA}{dt}$

$\Rightarrow \left.\frac{dr}{dt}\right|_{r=5} = \frac{1}{8\pi(5)} \cdot (-1)$

$\Rightarrow \left.\frac{dr}{dt}\right|_{r=5} = -\frac{1}{40\pi}$ cm/min.

The radius of the snowball is decreasing at a rate of $\frac{1}{40\pi}$ cm/min.

4.



We're given $\frac{d\theta}{dt}$.

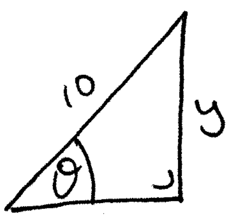
1 rev. every 2 minutes

$\Rightarrow \frac{1}{2}$ rev. every 1 minute

$\frac{1}{2}$ revolution $= \pi$ radians.

$\Rightarrow \frac{d\theta}{dt} = \pi$ rad/min.

We want $\left.\frac{dy}{dt}\right|_{y=6}$



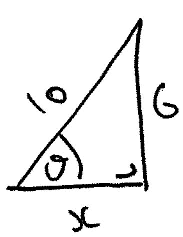
Formula relating y and θ

$$\sin\theta = \frac{y}{10}$$

$$\Rightarrow \quad y = 10\sin\theta$$

$$\Rightarrow \quad \frac{dy}{dt} = 10\cos\theta \frac{d\theta}{dt}$$

When $y = 6$, what is $\cos\theta$?



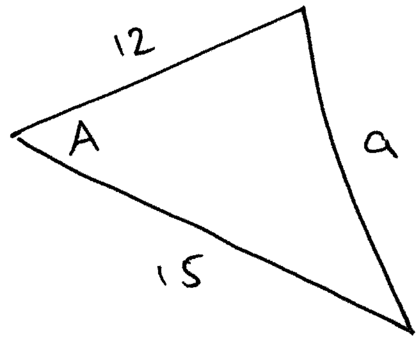
$$x^2 + 6^2 = 10^2 \quad \Rightarrow \quad x = 8$$

$$\Rightarrow \quad \cos\theta = \frac{8}{10} = \frac{4}{5}.$$

$$\left.\frac{dy}{dt}\right|_{y=6} = 10\left(\frac{4}{5}\right)\pi = 8\pi \text{ m/min}$$

The rider is rising at 8π m/min when he is 16m above the ground

5.



$2°/\text{min} = \frac{\pi}{90}$ rad/min

We're given $\frac{dA}{dt} = \frac{\pi}{90}$

We want $\left.\frac{da}{dt}\right|_{A=\frac{\pi}{3}}$.

$a^2 = b^2 + c^2 - 2bc\cos A \Rightarrow a^2 = (12)^2 + (15)^2 - 2(12)(15)\cos A$

$$\Rightarrow a^2 = 369 - 360\cos A.$$

$$\Rightarrow 2a\frac{da}{dt} = 360\sin A\frac{dA}{dt}$$

$$\Rightarrow \frac{da}{dt} = \frac{180}{a}\sin A\frac{dA}{dt}$$

When $A = \frac{\pi}{3}$, $a^2 = 369 - 360\cos\left(\frac{\pi}{3}\right) = 189$

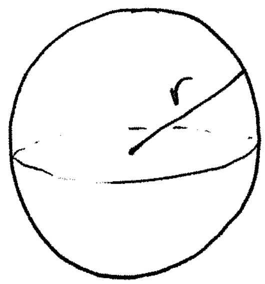
$$\Rightarrow a = \sqrt{189}$$

$$\Rightarrow \left.\frac{da}{dt}\right|_{A=\frac{\pi}{3}} = \frac{\cancel{180}^{\,2}}{\sqrt{189}}\sin\left(\frac{\pi}{3}\right)\frac{\pi}{\cancel{90}} = \frac{2\pi}{\sqrt{189}}\cdot\frac{\sqrt{3}}{\cancel{2}}$$

$$= \frac{\sqrt{3}\,\pi}{\sqrt{189}} = \frac{\sqrt{3}\,\pi}{3\sqrt{21}} = \frac{\cancel{\sqrt{3}}\,\pi}{3\cancel{\sqrt{3}}\sqrt{7}} = \frac{\pi}{3\sqrt{7}}$$

The third side is increasing at a rate of $\frac{\pi}{3\sqrt{7}}$ m/min when the angle between the other sides is $60°$.

6.



we're given $\frac{dV}{dt} = 5$

we want $\left.\frac{dr}{dt}\right|_{r=10}$.

$$V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

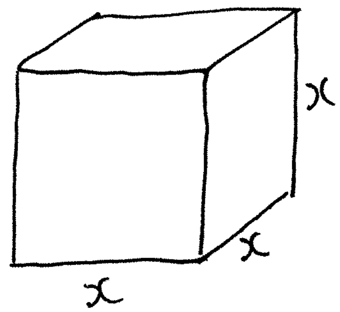
$$\Rightarrow \quad \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

$$\Rightarrow \quad \left.\frac{dr}{dt}\right|_{r=10} = \frac{1}{4\pi(100)}(5)$$

$$\Rightarrow \quad \left.\frac{dr}{dt}\right|_{r=10} = \frac{1}{80\pi} \text{ cm/sec}$$

The radius of the balloon is increasing at a rate of $\frac{1}{80\pi}$ cm/sec when the diameter is 20 cm

7.



We're given $\frac{dV}{dt} = -10$.

Surface area:

$$A = 6x^2.$$

We want $\left.\frac{dA}{dt}\right|_{x=10}$.

$$A = 6x^2 \Rightarrow \frac{dA}{dt} = 12x\frac{dx}{dt}.$$

We need $\frac{dx}{dt}$, but we have $\frac{dV}{dt}$.

$$V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{3x^2}\frac{dV}{dt}$$

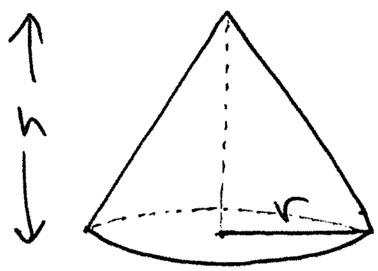
When $x = 10$, $\frac{dx}{dt} = \frac{1}{3(10)^2}(-10) = -\frac{1}{30}$

$$\Rightarrow \left.\frac{dA}{dt}\right|_{x=10} = 12(10)\left(-\frac{1}{30}\right) = \frac{-120}{30} = -4,$$

so $\left.\frac{dA}{dt}\right|_{x=10} = -4\ cm^2/min.$

The surface area is decreasing at a rate of $4cm^2/min$

8.



We're given $\frac{dV}{dt} = 20$

we want $\left.\frac{dh}{dt}\right|_{h=10}$.

$V = \frac{1}{3}\pi r^2 h$. We have $2r = h$

$\Rightarrow r = \frac{h}{2}$

$\Rightarrow V = \frac{1}{3}\pi \frac{h^2}{4} h$

$\Rightarrow V = \frac{1}{12}\pi h^3$,

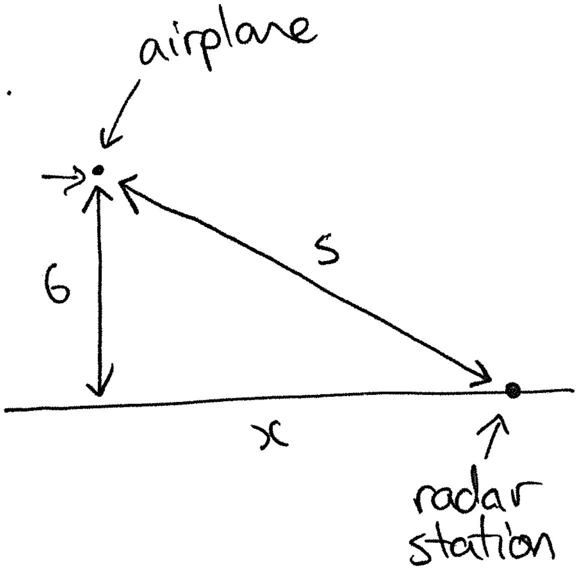
$$\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \frac{dh}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}$$

$$\Rightarrow \left.\frac{dh}{dt}\right|_{h=10} = \frac{4}{100\pi}(20) = \frac{4}{5\pi} \text{ m/min.}$$

The height of the pile is increasing at a rate of $\frac{4}{5\pi}$ m/min

9.



We're given $\frac{ds}{dt} = -400$

We want $\left.\frac{dx}{dt}\right|_{s=10}$

$$s^2 = x^2 + 6^2 \Rightarrow s^2 = x^2 + 36$$

$$\cancel{2}s\frac{ds}{dt} = \cancel{2}x\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}.$$

when $s = 10$ $\quad (10)^2 = x^2 + (6)^2$

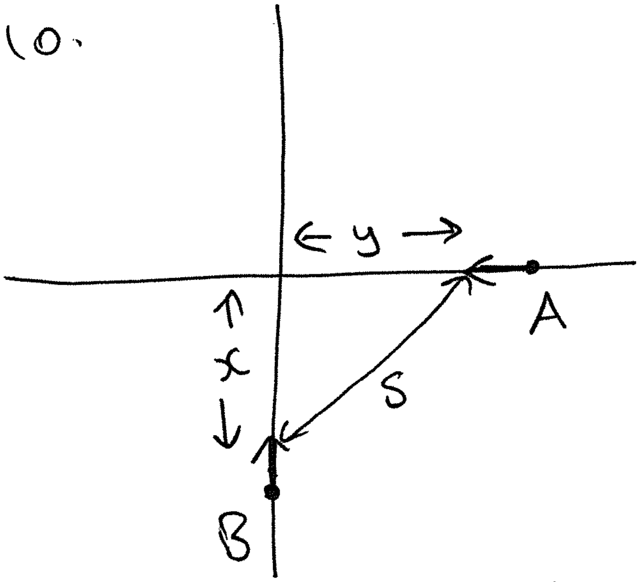
$$\Rightarrow x^2 = 64$$

$$\Rightarrow x = 8$$

$$\Rightarrow \left.\frac{dx}{dt}\right|_{s=10} = \frac{10}{8}(-400) = -500 \text{ km/h}.$$

The horizontal speed of the plane is 500 km/h.

10.



We're given

$$\frac{dx}{dt} = -30$$

and $$\frac{dy}{dt} = -20$$

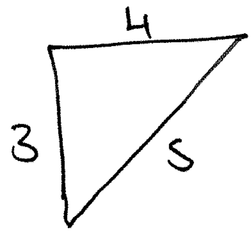
want $$\left.\frac{ds}{dt}\right|_{\substack{y=4\\x=3}}.$$

$$s^2 = x^2 + y^2 \Rightarrow \cancel{2}s\frac{ds}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$$

$$\Rightarrow s\frac{ds}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$\Rightarrow \frac{ds}{dt} = \frac{1}{s}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

when $x = 3$, $y = 4$



$s^2 = 3^2 + 4^2$

$s^2 = 25$

$s = 5$

$$\Rightarrow \left.\frac{ds}{dt}\right|_{\substack{x=3\\y=4}} = \frac{1}{5}\left(3(-30) + 4(-20)\right)$$

$$= \frac{1}{5}(-90-80) = -\frac{170}{5} = -34 \text{ m/hour}$$

The distance between the turtles is decreasing at a rate of 34 m/hour at that moment.