Math Help Centre

Differentiation Rules

Calculating a derivative using the definition can be quite tiresome. Luckily, unless you're specifically asked to use the definition, there are a number of derivative rules you can use to calculate derivatives. Here is a list of those rules:

- $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$.
- $\frac{d}{dx}(c) = 0$, where $c \in \mathbb{R}$.
- $\frac{d}{dx}(a \cdot f(x) + b \cdot g(x)) = a \cdot \frac{d}{dx}f(x) + b \cdot \frac{d}{dx}f(x)$, where $a, b \in \mathbb{R}$.
- $\frac{d}{dx}\left(e^x\right) = e^x$
- $\frac{d}{dx}\left(a^x\right) = a^x \ln(a)$
- $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$
- $\frac{d}{dx}(\log_b(x)) = \frac{1}{x \cdot \ln(b)}$

All of the above derivatives should be **learned** by all calculus students. The derivatives of the trig functions (below) should be learned by MATH1500 and MATH1510 students, but will not be required of MATH 1520 students.

- $\frac{d}{dx}(\sin(x)) = \cos(x)$
- $\frac{d}{dx}(\cos(x)) = -\sin(x)$
- $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
- $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
- $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
- $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$

Given that $f(x)$ and $g(x)$ are differentiable functions whose derivatives are represented by $f'(x)$ and $g'(x)$ respectively, the following rules also apply. **They must be learned.**

The Product Rule:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

The Quotient Rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f(x) - f(x)g'(x)}{\left(g(x)\right)^2}$$

The Chain Rule:

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$$

Example. For each of the following functions, find $f'(x)$

1. $f(x) = 7x^5 - 4x^9$.

$$f'(x) = 35x^4 - 36x^8$$

2. $f(x) = \sqrt[5]{x^7}$.

$$\begin{aligned} f(x) &= \sqrt[5]{x^7} = x^{\frac{7}{5}} \\ \implies f'(x) &= \frac{7}{5}x^{\frac{2}{5}} \end{aligned}$$

3. $f(x) = \pi^4$.

π^4 is a constant, so

$$f'(x) = 0$$

.

4. $f(x) = \dfrac{6}{x^4}$.

$$\begin{aligned} f(x) &= \frac{6}{x^4} = 6x^{-4} \\ \implies f'(x) &= -24x^{-5} = -\frac{24}{x^5} \end{aligned}$$

5. $f(x) = \log_4(x)$.

$$f'(x) = \frac{1}{x \cdot \ln(4)}$$

6. $f(x) = 7^x$.

$$f'(x) = 7^x \cdot \ln(7)$$

7. $f(x) = \tan(x) + \cot(x)$.

$$f'(x) = \sec^2(x) + \sec(x)\tan(x)$$

8. $f(x) = x^3 \cdot 3^x$.

Here, we need to use the product rule:

$$f'(x) = x^3 \cdot 3^x \ln(3) + 3^x \cdot 3x^2$$

9. $f(x) = \ln(x) \cdot \sqrt{x}$.

First, notice that $f(x) = \ln(x) \cdot \sqrt{x} = \ln(x) \cdot x^{\frac{1}{2}}$.

$$f'(x) = \ln(x) \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

10. $f(x) = \dfrac{\log_5(x)}{x^5 - 3x^4}$.

The quotient rule is required for this problem.

$$f'(x) = \frac{(x^5 - 3x^4) \cdot \dfrac{1}{x\ln(5)} - \log_5(x) \cdot (5x^4 - 12x^3)}{(x^5 - 3x^4)^2}$$

11. $f(x) = \dfrac{\ln(x)}{e^x}$.

$$f'(x) = \frac{e^x \cdot \dfrac{1}{x} - \ln(x)e^x}{(e^x)^2}$$

12. $f(x) = (x^4 - 3x^6)^3$.

The chain rule is required for this problem and the rest.

$$f'(x) = 3(x^4 - 3x^6)^2 \cdot (4x^3 - 18x^5)$$

13. $f(x) = \ln(x^3 + 6x^5)$.

$$f'(x) = \frac{1}{x^3 + 6x^5} \cdot (3x^2 + 30x^4)$$

14. $f(x) = \sin(\cos(x^2))$.

Here, the chain rule must be applied twice:

$$f'(x) = \cos(\cos(x^2)) \cdot (-\sin(x^2)) \cdot 2x$$

15. $f(x) = \sqrt{x^5 \ln(x)}$.

First, note that $f(x) = \sqrt{x^5 \ln(x)} = (x^5 \ln(x))^{\frac{1}{2}}$. Here we need to use the chain rule and product rule consecutively.

$$f'(x) = \frac{1}{2}(x^5 \ln(x))^{-\frac{1}{2}} \cdot \left(x^5 \cdot \frac{1}{x} + \ln(x) \cdot 5x^4\right)$$

16. $f(x) = \dfrac{\log_5 \cos(x^3 + x)}{e^{x^5+3x}}$.

For the problem, both the chain rule and the quotient rule are required.

$$f'(x) = \frac{e^{x^5+3x} \cdot \dfrac{1}{(x^3+x)\ln(5)} \cdot (3x^2+1) - \log_5 \cos(x^3+x) \cdot e^{x^5+3x} \cdot (5x^4+3)}{(e^{x^5+3x})^2}$$

Exercises

Differentiate each of the following functions.

Note: Some of these functions look like they require the product or quotient rule at a glance, but with a little manipulation, they aren't needed. Try to identify which ones.

1. $f(x) = \dfrac{3}{4}x^8$
2. $f(x) = \sqrt{x} - x$
3. $f(x) = (x^3 + 2x)e^x$
4. $f(x) = e^x(x + x\sqrt{x})$
5. $f(x) = \dfrac{x^2 + 2}{x^4 - 3x^2 + 1}$
6. $f(x) = \left(\dfrac{1}{x^2} - \dfrac{3}{x^4}\right)\left(x + 5x^3\right)$
7. $f(x) = \dfrac{x^3 - 2x\sqrt{x}}{x}$
8. $f(x) = (3x^2 + 2x)^7$
9. $f(x) = \sin(4x)$
10. $f(x) = \dfrac{(x^2 + 8)^4}{x + 2}$
11. $f(x) = (x^3 + 3)^6 \sin^5(3x^2)$