ERRATUM TO LIFTABLE MAPPING CLASS GROUP OF BALANCED SUPERELLIPTIC COVERS

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There is an error in the statement (but not the proof) of Lemma 6.3. It should be as follows.

Lemma 6.3. In the abelianization of $G_n$, $B^{n^2} = A^{-n^2-n}$.

This typo has a follow-on effect for the rest of the paper. The statement of Lemma 6.5 should be as follows, although the proof still holds as written.

Lemma 6.5. The abelianization $G_n/[G_n,G_n]$ admits the presentation
\[
\langle a, d, A, B \mid B^{n^2-n} = A^{1-n^2}, B^{n^2} = A^{-n^2-n}, a^2 = B, d^2 = A^{n+1}, T \rangle
\]
where $a = \phi(a_1)d = \phi(c), A = \phi(A_{12}), B = \phi(A_{13}),$ and $T$ is the set of all commutators.

Theorem 1.1 now has a different statement and proof, which we present now.

Theorem 1.1. Let $k \geq 3$. Then
\[
H_1(\text{LMod}_{p,k}(\Sigma_0, \mathcal{B})) \cong \begin{cases} 
\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z} & \text{if } n \text{ is odd}, \\
\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z} & \text{if } n \text{ is even.}
\end{cases}
\]

Proof. Replacing the generator $B$ with $a^2$ gives the presentation
\[
\langle a, d, A \mid a^{2n^2-2n} = A^{1-n^2}, a^{2n^2} = A^{-n^2-n}, d^2 = A^{n+1}, T \rangle
\]
for $G_n/[G_n,G_n]$. This presentation has presentation matrix
\[
\begin{bmatrix}
2 & -n - 1 & 0 \\
0 & n^2 - 1 & 2n^2 - 2n \\
0 & n^2 + n & 2n^2
\end{bmatrix}.
\]
If $n$ is odd we can perform row and column operations below to obtain the Smith normal form:
\[
\begin{bmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]
Therefore $G_n/[G_n,G_n] \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}$.

If $n$ is even, the Smith normal form will be:
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]
Therefore $G_n/[G_n,G_n] \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}$. \qed

Of course, the correct statement of Theorem 1.1 implies the following correct statement of Theorem 1.2.

**Theorem 1.2.** The abelianization of the balanced superelliptic mapping class group $H_1(\text{SMod}_{g,k}(\Sigma_g);\mathbb{Z})$ is an infinite non-cyclic abelian group. Furthermore, the first Betti number of $\text{SMod}_{g,k}(\Sigma_g)$ is 1.

We would like to thank Michael Lönne for pointing out the error in Lemma 6.3 and the subsequent incongruence in results.

**References**