2016 MANITOBA MATHEMATICAL CONTEST

For students in grade 12
9:00 AM – 11:00 AM
Wednesday, February 24, 2016

Sponsored by:

The Winnipeg Actuaries' Club

The Manitoba Association of Mathematics Teachers

The Canadian Mathematical Society

The University of Manitoba

Questions are found on both sides of this sheet. Answer as many as possible, but you are not expected to answer them all. **CALCULATORS ARE NOT PERMITTED**. Numerical answers by themselves, without explanation, will not receive full credit.

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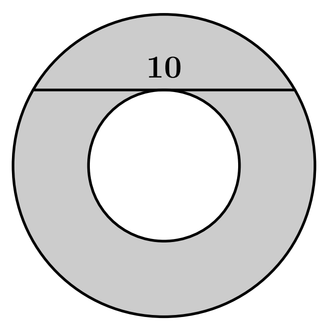
1. (a) What is the sum of the digits in the number $N = 2^8 5^4$?

   (b) Find the largest three digit number divisible by both 5 and 6.

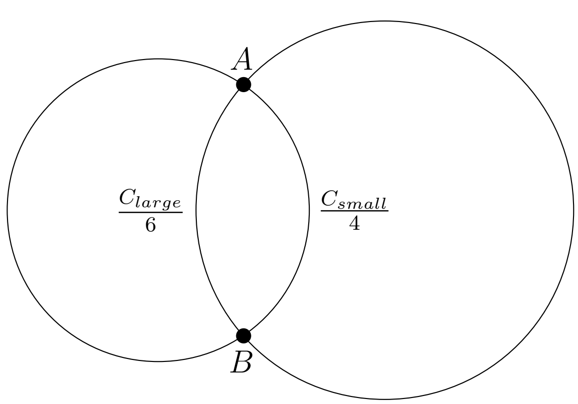
2. (a) A triangle initially has area 100 square units. If its base is increased by 10% and its altitude is decreased by 10% what is its new area?

   (b) What number is found two-thirds of the way from $\frac{1}{7}$ to $\frac{1}{5}$ on a number line?



3. (a) Two concentric circles have center O. A chord AB of the large circle tangent to the smaller circle has length 10. What is the area of the ring between the circles?

   (b)



Two circles intersect at points A and B. The length of arc $\overparen{AB}$ on the smaller circle is $\frac{1}{4}$ of the circumference. The length of the corresponding arc on the larger circle is $\frac{1}{6}$ of its circumference. If area of the smaller circle is 12, what is the area of the larger circle?

4. (a) The sum of two numbers is 10 and their product is 20. What is the sum of their cubes?
   (b) Can one make the expression $1 \star 2 \star 3 \star \cdots \star 10$ equal to 0 by replacing each instance of "$\star$" with either "+" or "−"?

5. Define an operation, $*$, for positive real numbers as follows:

$$a * b = \frac{ab}{a+b}.$$

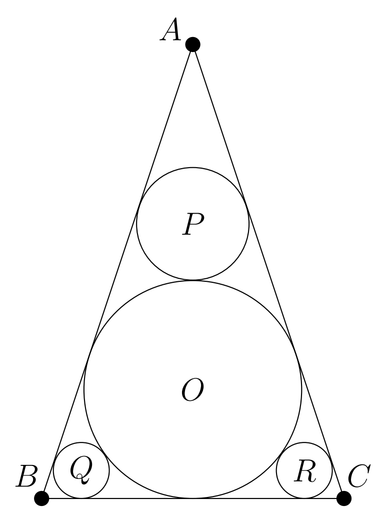
   (a) (3 marks) Verify that $(2 * 2) * 3$ is equal to $2 * (2 * 3)$.
   (b) (7 marks) Is it always the case that $a * (b * c) = (a * b) * c$? Justify your answer.

6. How many different positive integers n have the property that $n^2 - 2016$ is a perfect square? Find the smallest such number.

7. How many of the first 2016 positive integers are divisible by none of 7, 9 and 32?

8. Let $a_k = \frac{1}{\sqrt{2k-1}+\sqrt{2k+1}}$. Determine n so that $a_1 + a_2 + a_3 + \cdots + a_n = 8$.

9. Circle O is inscribed (as in the diagram) in isosceles triangle $\triangle ABC$ where $AB = AC$. Circles P, Q and R are each tangent to two sides of this triangle and, externally, to O as shown. The radius of O is 2 units and the radius of P is 1 unit. Find the radii of the other two circles.



10. Let x, y and z be real numbers satisfying $x + y + z = 15$ and $xy + yz + zx = 72$. Prove that $3 \leq x \leq 7$.