

2012 MANITOBA MATHEMATICAL CONTEST

For students in grade 12

9:00 AM – 11:00 AM

Thursday, February 23, 2012



Manitoba Association of
Mathematics Teachers

Sponsored by:

The Winnipeg Actuaries' Club

The Manitoba Association of Mathematics Teachers

The Canadian Mathematical Society

The University of Manitoba



Winnipeg
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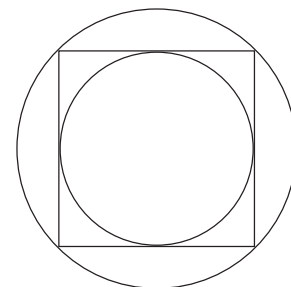
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Questions are found on both sides of this sheet. Answer as many as possible, but you need not complete the paper. **CALCULATORS ARE NOT PERMITTED.** Numerical answers by themselves, without explanation, will not receive full credit.

- Find three consecutive even integers with a sum of 36.
 - Find three consecutive integers so that smallest plus twice the largest is 15 more than the middle integer.
- Find the number half way between $\frac{2}{5}$ and $\frac{5}{8}$.
 - What single percentage increase is equivalent to a series of two increases of 10%?
- A circle has an area of 5 square units. A square is constructed around the circle so that each side of the square is tangent to the circle. A larger circle is constructed around the square so that it passes through all four vertices of the square. Find the area of the larger circle.
 - A rectangular tank has length 2 metres, width 3 metres, and height 3 metres and is full of water. A cylindrical tank has radius 2 metres and height 3 metres, and is empty. Water is siphoned from the rectangular tank to the cylindrical tank. The siphon will stop flowing when the depth of water is the same in both tanks. What will be the depth at that moment?



4. (a) The digits 1, 2, 3, 4 can be arranged to form a four digit number in 24 different ways. What is the sum of these 24 four digit numbers?
 (b) Write $x^4 + 4$ as the product of two quadratic polynomials with integer coefficients.
5. (a) Find the value of x for which $\sqrt{x} - 1$ and $\sqrt{x} + 1$ are reciprocals.
 (b) Prove that there is no real value of x for which x and $1 - 2x$ are reciprocals.
6. To represent a number in a base system other than 10, say base $B > 0$, let us write " S_B " where S is a string of suitable digits in base B (that is, elements of the set $\{0, 1, 2, \dots, B - 1\}$). (If no base is given it is understood that a number is written in base 10.) Thus, $112_3 = 1 \cdot 3^2 + 1 \cdot 3 + 2 = 14$ and $1036_7 = 1 \cdot 7^3 + 0 \cdot 7^2 + 3 \cdot 7 + 6 = 370$.
 (a) In what base B is 213_B equal to 58?
 (b) Evaluate the following expression. Report your answer in base 10.

$${}^{2012} \left({}^{22} \left({}^{12} \left({}^{21} \left({}^{11} \right) \right) \right) \right)$$

7. In the polynomial equation

$$x^7 + 3x^6 + 2x^5 + 4x^4 + 12x^3 + x^2 + x - 2 = (x^2 + 3x + 2)q(x) + ax + b,$$
 find the values of a and b .
8. A line with positive slope passes through the origin and is tangent to the circle $(x - 5)^2 + y^2 = 9$. Find the equation of this line.
9. Let A be the point $(4, 9)$ and B be the point $(10, 5)$. A very small insect starts at A , crawls to a point C on the x -axis, and then crawls to the point B . If he took the shortest possible path to do this, find the distance travelled and the co-ordinates of the point C .
10. A square of area 1 is divided into three rectangles which are geometrically similar (i.e., they have the same ratio of long to short sides) but no two of which are congruent. Write A , B and C for the areas of the rectangles, ordered from largest to smallest. Prove that $(AC)^2 = B^5$.