2008 MANITOBA MATHEMATICAL COMPETITION
QUESTIONS

1. (a) Solve the equation $x + \frac{6}{x+1} = 4$.

   (b) Solve the equation $x^5 + 36x = 13x^3$.

2. (a) Find the real numbers a and b if 2 and 3 are roots of $x^3 + ax^2 + bx + 6 = 0$.

   (b) In this problem A and B are the two points at which the graph of the equation $x^2 + y^2 = 8$ meets the graph of the equation $y = |x|$. What is the length of the segment AB?

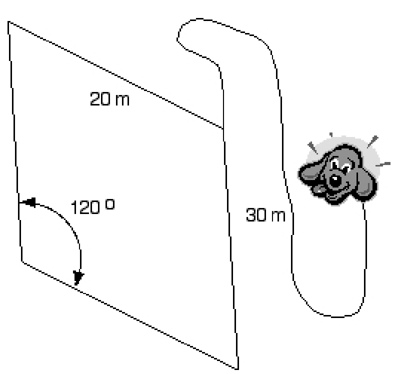
3. (a) Find an equation of the circle passing through the origin and the points with coordinates $(10, 0)$ and $(0, 8)$.

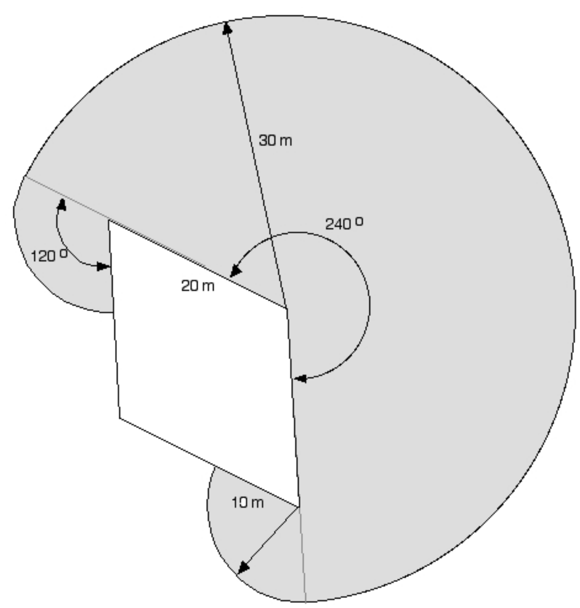
   (b) Find an equation of the line tangent to the circle with equation $(x-2)^2 + (y+1)^2 = 25$ at the point with coordinates $(5, 3)$.

4. (a) In this problem c and d are real numbers. The point on the graph of the equation $y = x^2 + cx + d$ which is nearest to the x-axis is $(-2, 5)$. find the values of c and d.

   (b) Car A is travelling due west at a constant speed of 50 km/hr. Car B is travelling due east at a constant speed of 60 km/hr. At 1:00 p.m. car A is 40 km due north of car B. At 2:00 p.m. what is the distance between the two cars (as the crow flies)?

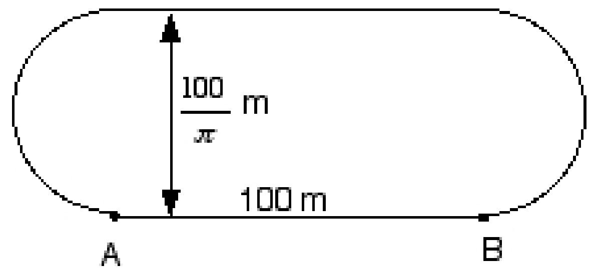
5. A fenced property has the shape of a rhombus, as in the figure. The length of each side of the rhombus is 20 m. A dog outside the property is tethered to one corner of the rhombus as shown in the diagram. If the dog's leash is 30 m long, how large an area can the dog cover?





6. A race track is built with two straight parallel sides and semicircles at the ends (as in the figure). The parallel sides are 100 m long and $\frac{100}{\pi}$ m apart. Runner Alpha at position A starts running

clockwise around the track at 2 m/sec. At this precise moment a second runner Beta enters the track at position B which is 100 m from position A, running at 5 m/sec. If Beta wants to meet Alpha as soon as possible, should he run clockwise or counterclockwise around the track to achieve his goal?



7. For what values of x does $\frac{1}{x+1} + \frac{1}{2x} > 1$ hold?

8. In this problem x, y and z are real numbers. Find all possible values of a if:

$$a = \frac{x}{|x|} + \frac{y}{|y|} + \frac{z}{|z|}.$$

9. Prove that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

10. In the diagram $\triangle ABC$ is isosceles with $AB = AC$. Prove that if $LP = PM$, then $LB = CM$.