2023 MANITOBA MATHEMATICAL COMPETITION

(for students in grade 12)

Wednesday February 22, 2023, 9 – 11 AM

Sponsors:

M.A.M.T. Manitoba Association of Mathematics Teachers

Winnipeg Actuaries' Club

University of Manitoba

CMS SMC

☐ Mr.
☐ Mrs.
☐ Ms.

↑ Student's Name (print). Underline the Surname ↑

↑ Student's Signature ↑

↑ Student's School ↑

↑ Student's Home Street Address (or P. O. Box #) ↑

↑ City/Town Postal Code ↑

↑ Email ↑

Instructions for participants: Before the contest begins, complete the above information. Put no personal identifying information on any other pages. You should have received 12 pages in total, including this page, all showing the same paper number.

Answer each question on the page where it appears. If you run out of space below a question work may be continued on the back of the same page; if that is insufficient you may further continue work on blank pages 23 and 24.

No for-credit work should appear on the back of this cover page (it may be used for scrap).

Don't refer in any solution to work done on other questions; they are marked independently.

No aids are permitted—no straight edges, compasses or other mechanical drawing devices, electronics (cell phones, electronic watches, translators, tablets, calculators etc.).

This space may be used for scratch work. Do not continue solutions on this page—no credit will be given for work appearing here.

Question 1

(a) If $x = 4$, find the value of $\sqrt{x + \sqrt{6x + 1}}$.

(b) Find a pair of distinct positive integers x and y such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$.

This space may be used to continue your solution for Question 1; it may be further continued on page 23 or 24—see instructions on page 23.

Question 2

(a) A set of consecutive integers has an average of 37. The third number and the sixth number, in sequence, have a sum of 19. Find the largest of the numbers.

(b) Susan wrote two consecutive positive integers on a piece of paper. She then multiplied each of them by two and squared the results. The difference between the resulting squares was 84. What were the original numbers?

This space may be used to continue your solution for Question 2; it may be further continued on page 23 or 24—see instructions on page 23.

Question 3

(a) Azeen drove his car for 160 km. For the first 40 km he travelled at an average speed of 50 km/h. For the rest of the trip he averaged 100 km/hr. What was his average speed for the entire trip?

(b) Glenda wrote three mathematics tests and had marks over 50 on all of them. She calculated her average mark and got a result of 48. She knew this could not be right. When she inspected her work she discovered that she had exchanged the two digits of her lowest mark. She then correctly determined her average to be 57. What was her lowest mark?

This space may be used to continue your solution for Question 3; it may be further continued on page 23 or 24—see instructions on page 23.

Question 4

(a) Find all ordered pairs (m, n) where m and n are integers satisfying $(m-2)^2(n+1) = 72$.

(b) The digits 1, 2, 3, 4 can be arranged (without repetitions) to form 24 different four digit numbers. What is the sum of those numbers?

This space may be used to continue your solution for Question 4; it may be further continued on page 23 or 24—see instructions on page 23.

Question 5

In each case, solve for x.

(a) $\dfrac{\sqrt{2}-1}{\sqrt{x}-1} = \dfrac{\sqrt{x}-1}{\sqrt{2}+1}$

(b) $\left(x^2-3x\right)^2 - 8 = 2\left(x^2-3x\right)$

This space may be used to continue your solution for Question 5; it may be further continued on page 23 or 24—see instructions on page 23.

Question 6

If p and q are odd integers, prove that

$$x^2 + 2px + 2q = 0$$

has no rational roots.

This space may be used to continue your solution for Question 6; it may be further continued on page 23 or 24—see instructions on page 23.

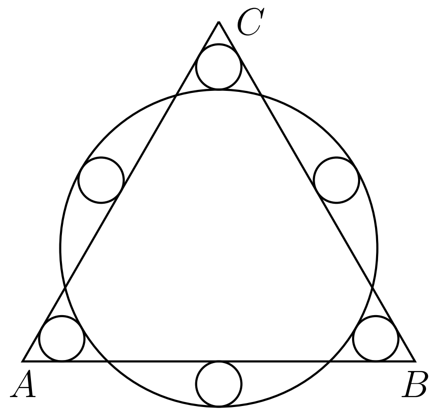
Question 7

A corridor has seven lights numbered 1 to 7, each with its own switch. Initially, all lights are on. Adam makes a random selection of three lights which he turns off. Betty makes a random selection of three lights and changes their status (if a light was on she turns it off and vice versa). Chris then makes a random selection of three lights and changes their status. What is the probability that all the lights have been thus turned off?

This space may be used to continue your solution for Question 7; it may be further continued on page 23 or 24—see instructions on page 23.

Question 8

Six small circles each have radius of one unit. Each is tangent to a larger circle as shown, three of them also tangent to two sides of an equilateral triangle $\triangle ABC$, and the other three each tangent to a side of $\triangle ABC$ at its midpoint. Find the radius of the larger circle.



This space may be used to continue your solution for Question 8; it may be further continued on page 23 or 24—see instructions on page 23.

Question 9

For how many positive integers $n \leq 100$ is the fraction $\dfrac{17n+4}{4n+3}$ reducible?

(Note: A fraction is reducible if it is is equivalent to another fraction with smaller denominator. For example, $\dfrac{4}{6}$ is reducible because it is equivalent to $\dfrac{2}{3}$ whereas $\dfrac{5}{6}$ is not reducible.)

This space may be used to continue your solution for Question 9; it may be further continued on page 23 or 24—see instructions on page 23.

Question 10

The edges of a rectangular solid are all of integer lengths. Let p and q be odd prime numbers. Exactly five such solids have a volume of $2pq$; their dimensions are $2 \times p \times q$, $1 \times 2p \times q$, $1 \times p \times 2q$, $1 \times 2 \times pq$ and $1 \times 1 \times 2pq$.

(a) How many possible solids are there if the volume is to be $4pq$?

(b) How many possible solids are there if the volume is $2^{2n}pq$ where n is a positive integer? Express your answer in terms of n.

This space may be used to continue your solution for Question 10; it may be further continued on page 23 or 24—see instructions on page 23.

Both sides of this sheet may be used for continuation of solutions or for scratch work.

To receive credit for work continued here:

1. Clearly indicate in your solution that it is continued here.
2. Clearly indicate here which question is being continued (e.g., "Q7 (cont.)").
3. Clearly separate continued work from different questions and from scratch calculations.

This space may be used for scratch work, or to continue solutions—see instructions on page 23