Questions are found on both sides of this sheet. Answer as many as possible, but you are not expected to answer them all. **CALCULATORS ARE NOT PERMITTED.** Numerical answers by themselves, without explanation, will not receive full credit.

1. (a) What is the probability that a randomly chosen number between 1 and 50 inclusive will have a 4 as a digit?

(b) Let \( X = |(-100) \cdot (-99) \cdots (99) \cdot (100)| \) (the absolute value of the product of all the integers from \(-100\) to 100), and let \( Y = (-100)^2 + (-99)^2 + \cdots + 99^2 + 100^2 \). Which number is larger, \( X \) or \( Y \)? (mention a reason).

2. Find the area enclosed by the graph of
   (a) \( x^2 + y^2 = 9 \)
   (b) \(|x| + |y| = 3\)

3. (a) If \( y \) is 2 more than \( x \), and
   \[ z = \frac{y}{x} + \frac{x}{y} + 2 \]
   
   then how much more than \( x \) is \( z \)?

(b) Solve for \( x \):
   \[ \frac{x^3 + x - 2}{x - 1} = 5 \]

4. (a) Apollo runs in a race with 10 runners where 5 distinct trophies (1st place, 2nd place etc.) are given to the top 5 winners, in how many different ways can the prizes be given if there are no ties, and Apollo must be one of the top 3 winners?

(b) If \( n \) is a positive integer and \( 2n + 1 \) is a perfect square, show that \( n + 1 \) is the sum of two consecutive perfect squares.
5. Solve the equation
\[ \sqrt{x} + 3\sqrt{x} = 4\sqrt{x}. \]

6. Consider the set of numbers \( x \) satisfying \( 0 \leq x \leq 1 \). Two numbers are randomly chosen from this interval. What is the probability that they differ by less than 0.2?

7. A wheel with radius 2 meters rolls around the outside of a regular convex heptagon* of side length 3 meters. Determine the length of the path traced out by the centre of the wheel.

*regular heptagon: a 7-sided polygon with equal sides and angles.

8. \( \square ABCD \) is a square with sides of length 2. At each vertex a quarter circle of radius 2 is drawn as shown. Find the area of the intersection, \( X \), of the four semicircles.

9. On each side of a convex* quadrilateral with area \( Q \) a square is constructed. Segments are added between near vertices of neighbouring squares, creating a figure consisting of \( Q \), surrounded by four squares, alternating in cyclic order with four triangles of areas \( A \), \( B \), \( C \) and \( D \) (see diagram). Prove that \( A + C = B + D = Q \).

*convex: \( R \) is a convex region if any line segment whose endpoints are contained in \( R \) is entirely contained in \( R \).

10. Find all ordered triples of polynomials \( p(x), q(x), r(x) \) so that the following conditions hold:

- \( p(x)q(x)r(x) = (1 + x + x^2 + x^3 + x^4 + x^5)^3 \)
- \( p(1) = q(1) = r(1) \)
- \( \deg(p) \leq \deg(q) \leq \deg(r) \)  
  \( (\deg(f) \) denotes the degree of polynomial \( f(x) \) \)