1. (a) Solve for $x$:

\[
\frac{5}{x} - \frac{7 + 2x}{3x} = 3
\]

(b) Find the ratio of $x$ to $y$, given that

\[
\frac{x - y}{x + y} = \frac{2}{5}
\]

2. (a) Let “$*$” denote an operation defined as follows: $a * b = 2ab - b^3$. Find the value of $(4 * 3) * 2$.

(b) The original price of an item is reduced by 20%. A month later its reduced price is doubled. At the end of the year it is sold in a discount sale. What exact percentage discount was applied if the sale price was equal to the original price?

3. In a Fibonacci-like sequence each term is the sum of the two terms immediately preceding it. For example the Fibonacci sequence $1, 1, 2, 3, 5, 8, \ldots$ and the Lucas sequence $1, 3, 4, 7, 11, 18, \ldots$ are both Fibonacci-like sequences.

(a) The first term of a Fibonacci-like sequence is 4 and the fifth term is 2. Find the second, third and fourth terms.

(b) If $x, y, 2x - 1, x + 4$ is a Fibonacci-like sequence, find the values of $x$ and $y$. 
4. (a) Find, with justification, all pairs of real numbers \((a, b)\) such that 
\[a^2 + 2ab + b^2 = 9\] and 
\[a - b = 5.\]

(b) Find, with justification, all pairs of real numbers \((a, b)\) such that 
\[a^2 - ab + b^2 = 0.\]

5. (a) Find all possible values of \(a\) such that the line with equation \(x + y = a\) is tangent to the circle with equation \(x^2 + y^2 = 25.\)

(b) One side of an equilateral triangle is the 12 cm diameter of a semi-circle, as shown. Find the area of the portion of the triangle that is outside the semi-circle.

6. The sides of a right triangle are \(x + 2\), \(x + 6\) and \(2x\). Find all possible values of \(x\).

7. \(ABCD\) is a rectangle. \(E\) is on \(AB\). \(CE\) intersects diagonal \(DB\) at \(F\). \(\triangle ADE\) has area 50. \(\triangle EFB\) has area 40. Find the area of rectangle \(ABCD\).

8. The lines \(y = -2x + 12\), \(x = 2\) and \(y = mx\) intersect to form a triangle with area 2. Find all possible values for \(m\).

9. Let \(A, B, C\) be vertices of an equilateral triangle, with \(D\) an interior point such that the lengths of \(AD\), \(BD\), \(CD\) are 3, 4 and 5, respectively. What is the area of the triangle?

10. The expression \(n!\) denotes the product \(1 \cdot 2 \cdot 3 \cdots n\) and is read as “\(n\) factorial”.
For example \(5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120.\)

(a) The product \((2!)(3!)(4!)(5!)(6!)(7!)(8!)(9!)(10!)(11!)(12!)\) can be written in the form \(M^2N!\) where \(M, N\) are positive integers. Find a suitable value of \(N\) and justify your answer.

(b) Prove that, for every \(n \geq 1\), \((2!)(3!)(4!) \cdots ((4n)!)\) can be written as the product of a square and a factorial.