Manitoba Mathematical Competition
QUESTIONS ONLY

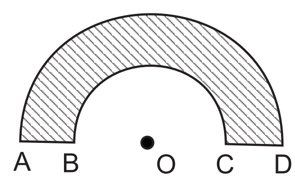
Draft Date: February 16, 2010

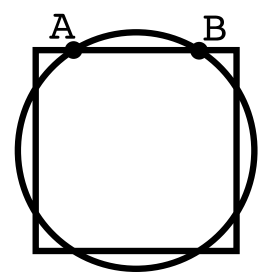
1. (a) If $x^2 - y^2 = 39$ and $x + y = 3$, find $x - y$.
   (b) Solve for x: $x + \sqrt{x} = 20$.

2. (a) A performer asks the members of his audience to think of a number. They are to increase this number by 3. The result is to be multiplied by 2. 10 is subtracted from the new result. This latest result is divided by 2. When told the final result an audience member obtains, he immediately tells them their original number. What one-step formula can he use to convert the final result back into the original number?
   (b) A fair coin is tossed four times. What is the probability that it shows heads three times and tails only once?

3. (a) At what points does the circle with equation $x^2 + y^2 = 3$ intersect the parabola with equation $y = 2x^2$?
   (b) The three lines whose equations are $y = x - 7$, $x + y = 3$ and $y = kx + 8$ pass through a common point. Find the value of k.

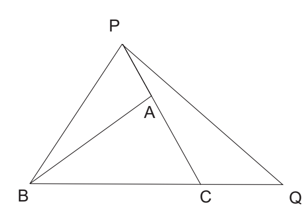
4. (a) Two semicircles, as shown, have centre O. Points A, B, O, C and D are colinear with $AB = CD = 1$. The shaded area is 15π. Find the length of AD.





   (b) Consider a circle and a square whose areas are equal and which have the same centre of symmetry (see diagram). If the radius of the circle is 2, find the length of AB. (Express your answer in terms of π.)

5. (a) The number 64 is both a perfect square and a perfect cube. Find the next positive integer with this property.
   (b) How many solutions of $2x + 3y = 763$ are there in positive integers x, y?



6. In the diagram, $AP = \frac{1}{3}PC$ and $CQ = \frac{1}{2}BC$. Prove that the area of $\triangle BPA$ is two-thirds times the area of $\triangle CPQ$.

7. Five distinct integers are added in pairs, giving the ten sums

$$7, 11, 12, 13, 14, 18, 21, 22, 26, 28.$$

Find the numbers, justifying your answer with a series of deductions clearly demonstrating that there is no other possibility.

8. A line with slope 1 meets the parabola $y = x^2$ at A and B. If the length of segment AB is 3 what is the equation of that line?

9. Solve for x and y:

$$\begin{aligned} x + y + xy + 2 &= 0 \\ x^2 + y^2 + x^2y^2 - 16 &= 0 \end{aligned}$$

10. All three sides of a right triangle are integers. Prove that the area of the triangle:

    (a) is also an integer;

    (b) is divisible by 3;

    (c) is even.