2025 MANITOBA MATHEMATICAL COMPETITION

(for students in grade 12)

Wednesday, February 26, 2025, 9 – 11 AM

Sponsors:

☐ Mr.
☐ Mrs.
☐ Ms.

↑ Student's Name (print). Underline the Surname ↑

↑ Student's Signature ↑

↑ Student's School ↑

↑ Student's Home Street Address (or P. O. Box #) ↑

↑ City/Town Postal Code ↑

↑ Email ↑

Instructions for participants: Before the contest begins, complete the above information. Put no personal identifying information on any other pages. You should have received 12 pages in total, including this page, all showing the same paper number.

Answer each question on the page where it appears. If you run out of space below a question work may be continued on the back of the same page; if that is insufficient you may further continue work on blank pages 23 and 24.

No for-credit work should appear on the back of this cover page (it may be used for scrap).

Don't refer in any solution to work done on other questions; they are marked independently.

No aids are permitted—no straight edges, compasses or other mechanical drawing devices, electronics (cell phones, electronic watches, translators, tablets, calculators etc.).

This space may be used for scratch work. Do not continue solutions on this page—no credit will be given for work appearing here.

Question 1

(a) Solve for x:

$$2^{3x}8^{x+1} = 2^{3^5}$$

(b) Solve for x:

$$2^{3x} + 8^{x+1} = 12^2$$

This space may be used to continue your solution for Question 1; it may be further continued on page 23 or 24—see instructions on page 23.

Question 2

(a) Find all solutions to the equation

$$x^2 - 4x + y^2 + 6y + 13 = 0$$

(b) Define polynomial

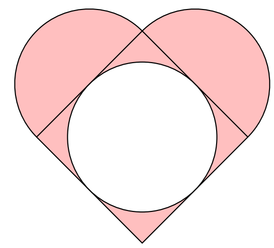
$$p(x) = ax^2 + bx + c$$

where a, b, c are real numbers. Find all possible triples a, b, c, if any, such that $p(1) = 3$, $p(2) = 15$ and $p(3) = 37$.

This space may be used to continue your solution for Question 2; it may be further continued on page 23 or 24—see instructions on page 23.

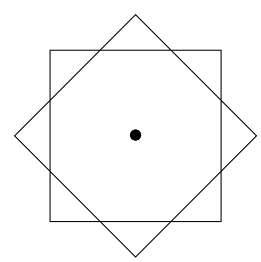
Question 3

(a) Dominic, who works with silver, plans to impress Marci with a handmade heart-shaped pendant. He starts with a 3 cm $\times$ 3 cm square, hung at 45 degrees. A semicircle is welded along each of two adjacent sides to form lobes. The whole item is anodized with a red coating except for a circle inscribed in the original square (see diagram).



What is the area of the red portion on the face of the finished pendant?

(b) Two congruent squares share a common centre point with one rotated at 45° (See diagram). What fraction of original area of each square overlaps the other square? Give your answer in the form $\frac{a+b\sqrt{c}}{d}$ where a, b, c, d are integers.



This space may be used to continue your solution for Question 3; it may be further continued on page 23 or 24—see instructions on page 23.

Question 4

For positive integers n, let us write $n!$ for the product $1 \cdot 2 \cdot 3 \cdots n$. For example, $3! = 1 \cdot 2 \cdot 3 = 6$ and $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$.

(a) Notice that $3! \cdot 4! = 144 = 12^2$

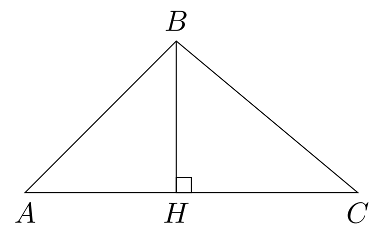
Find another pair of <u>distinct</u> positive integers m, n, both greater than 1, such that $m! \cdot n!$ is a perfect square.

(b) How many distinct positive divisors does $10!$ have?

This space may be used to continue your solution for Question 4; it may be further continued on page 23 or 24—see instructions on page 23.

Question 5

(a) In the triangle ABC shown, $\angle ABC = 105°$, BH is perpendicular to AC, and $BH = AH = 2$. Find the area of the triangle ABC.



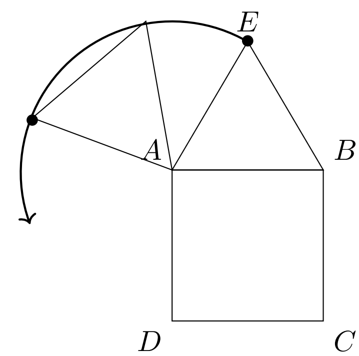
(b) Nuno had a craft box of sticks each of whose lengths is a positive integer. Taking groups of three sticks he forms triangles (excluding degenerate triangles—arrangements of three vertices lying in a single line) with sticks for sides.

Eventually he is left with 10 sticks among which no three form a triangle. The smallest has length 1 cm. What is the minimum length of the longest of the remaining 10?

This space may be used to continue your solution for Question 5; it may be further continued on page 23 or 24—see instructions on page 23.

Question 6

Square $\square ABCD$ has side length 1 m. An equilateral triangle $\triangle ABE$ shares side AB, but can move freely (See diagram). The triangle rolls freely around the square counterclockwise—thus it begins by rotating around A until its side AE coincides with side AD of the square. Then it rotates around D in a similar fashion, and so on.



Eventually the vertex at E returns to its original position. How long is the figure traced out by this vertex in the process?

This space may be used to continue your solution for Question 6; it may be further continued on page 23 or 24—see instructions on page 23.

Question 7

You are given that the following expression,

$$\sqrt{3+\sqrt{33+\sqrt{3+\sqrt{33+\sqrt{3+\cdots}}}}}$$

approaches some number (meaning that if one begins with $\sqrt{3}$, adds 33, extracts square root, then adds 3 and extracts square root to obtain $\sqrt{3+\sqrt{33+\sqrt{3}}}$, and continues iterating this process, the result each time gets closer and closer to some number, eventually approaching within any specified distance).

Find that number, justifying your answer (you needn't justify the statement given above).

This space may be used to continue your solution for Question 7; it may be further continued on page 23 or 24—see instructions on page 23.

Question 8

Andrii caught some fish. He gave the three largest fish to his dog, reducing the total weight of the catch by 35%. Then he gave the three smallest fish to his cat, reducing the remaining weight by 5/13. The rest of the fish were eaten by his family for dinner. How many fish did Andrii catch?

This space may be used to continue your solution for Question 8; it may be further continued on page 23 or 24—see instructions on page 23.

Question 9

Jack likes the numbers that are divisible by 3 and Jill likes the numbers that do not have digits $0, 3, 6, 9$. For example, both Jack and Jill like 48 and 1212.

(a) How many two-digit numbers are liked by both Jack and Jill?

(b) How many three-digit numbers are liked by both Jack and Jill ?

(c) If a_k denotes the number of k-digit numbers liked by both Jack and Jill, find the smallest k, such that a_k is divisible by 10.

This space may be used to continue your solution for Question 9; it may be further continued on page 23 or 24—see instructions on page 23.

Question 10

Let us call an integer N **nice** if it has no repeated digits and the number obtained by removing the first digit of N is equal to $\frac{N}{5}$.

(a) Find at least one nice number.

(b) Find all nice numbers.

This space may be used to continue your solution for Question 10; it may be further continued on page 23 or 24—see instructions on page 23.

Both sides of this sheet may be used for continuation of solutions or for scratch work.

To receive credit for work continued here:

1. Clearly indicate in your solution that it is continued here.
2. Clearly indicate here which question is being continued (e.g., “Q7 (cont.)”).
3. Clearly separate continued work from different questions and from scratch calculations.

This space may be used for scratch work, or to continue solutions—see instructions on page 23