2011 MANITOBA MATHEMATICAL CONTEST

For students in grade 12
9:00 AM – 11:00 AM
Thursday, February 24, 2011

Manitoba Association of Mathematics Teachers

Winnipeg Actuaries' Club

Sponsored by:

The Winnipeg Actuaries' Club

The Manitoba Association of Mathematics Teachers

The Canadian Mathematical Society

The University of Manitoba

UNIVERSITY OF MANITOBA

Canadian Mathematical Society

Answer as many questions as possible, but you need not complete the paper. **HAND CALCULATORS ARE NOT PERMITTED**. Numerical answers by themselves, without explanation, will not receive full credit.

---

1. (a) A square has a diagonal 10 cm long. Find the area of the square.

   (b) $ABCD$ is a square. AB and CD are increased by 10% while BC and DA are decreased by 10%. By what percent does the area change?

2. On a number line the point A is at $\frac{1}{5}$ and the point B is at $\frac{1}{3}$.

   (a) The point C is half way between A and B. Where is C located?

   (b) Point D is to the right of B. If $BD = 2(AB)$, find the location of D.

3. (a) Solve for x: $\frac{1}{4}\left(2^{4x}\right) = 32$

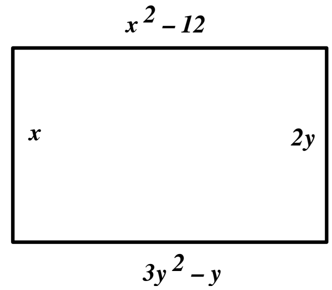
   (b) Solve for x and y:

$$\begin{aligned} 2^x - y &= 7 \\ 2^{x+1} + 2y &= 2 \end{aligned}$$

4. (a) Let n be the product of the first 30 positive integers. In how many zeros does the base ten representation of n end?

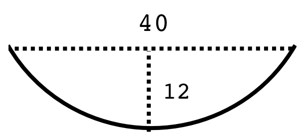
   (b) Find a pair of positive integers (a, b) that satisfies $a^2 - b^2 = 2011$.

5. (a) The sides of a rectangle are as shown. Find a numerical value for the area of the rectangle.

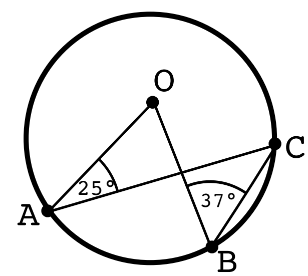


(b) Prove that, for every positive integer n, the number $n^3 + 2n$ is divisible by 3.

6. (a) A beach ball, floating on a lake, was not removed until the water had frozen. It left an impression in the ice that was 12 cm deep and 40 cm across. What was the radius of the beach ball?



(b) O is the centre of the circle in the diagram, $\angle OAC = 25°$ and $\angle OBC = 37°$. Find $\angle AOB$.



7. $56a = 65b$, where a, b are positive integers. Prove that $a+b$ is composite.

8. Find the least positive integer n for which $\dfrac{n-11}{3n+8}$ is a nonzero reducible fraction (i.e., not in lowest terms).

9. A function f satisfies the equation $f(x+1) = \frac{2f(x)+x}{3} + 1$ for all real numbers x. Suppose $f(1000) = 2011$. Find the value of $f(2011)$.

10. There exist positive integers whose value is quadrupled by moving the rightmost decimal digit into the leftmost position. Find the smallest such number.