
Manitoba Mathematical Competition
QUESTIONS

1. (a) The average of x and y is 3. What is the value of z if $3x - z = z - 3y$?

(b) If real numbers a and b satisfy both $a - b = 3$ and $ab = 2$, what is the value of $\frac{1}{a} - \frac{1}{b}$?

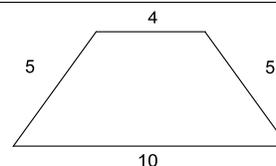
2. (a) Find the area bounded by the x -axis, the y -axis and the line $5x + 4y = 20$.

(b) A circle has diameter AB , where A is the point $(3, 5)$ and B is the point $(5, 9)$. A line through the origin divides this circle into two regions of equal area. Find the slope of that line.

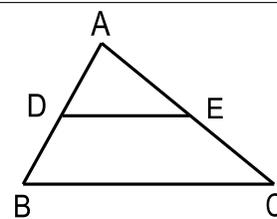
3. (a) Solve for x : $\sqrt{3\sqrt{3}} = 3^x$.

(b) Solve the equation $x^5 - 5x^3 + 4x = 0$.

4. (a) An isosceles trapezoid has parallel sides of length 4 and 10, as in the diagram. Find its area.



(b) In $\triangle ABC$, D is the midpoint of AB and E is the midpoint of AC . If $\triangle ADE$ has an area of 4, what is the area of trapezoid $DECB$?



5. (a) Given that numbers x, y and z satisfy the equations

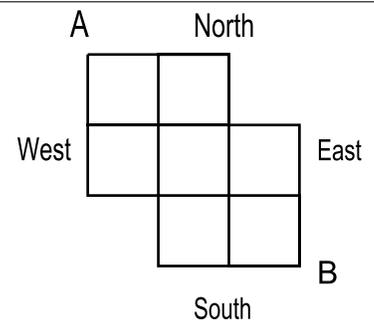
$$x + 2y + 3z = 2008 \text{ and } 3x + 2y + z = 8002,$$

what is the value of $x + y + z$?

- (b) Solve the equation $(x^2 - 3x + 2)^2 + (x^2 - 4x + 3)^2 + (x^2 - 5x + 4)^2 = 0$.

6. Find the sum of the digits of the number $10^{100} - 10^8 - 3$.

7. A traveller at A wishes to reach B . To get there he must walk six blocks, travelling only on the streets shown in the diagram. How many possible routes are there?



8. Solve for x, y and z :

$$\begin{aligned}x + y + z &= 4 \\x - y + z &= 0 \\x^2 + y^2 + z^2 &= 14\end{aligned}$$

9. A, B and C are points on a circle of radius 1 such that $AB = \sqrt{2}$ and $\angle ABC = 60^\circ$. Find AC .

10. 2009 points are chosen on the line AB all lying outside the segment AB . Prove that the sum of the distances from these points to the point A is not equal to the sum of their distances to point B .