MANITOBA MATHEMATICAL CONTEST, 2003

For Students in Senior 4

9:00 AM – 11:00 AM

Wednesday, February 19, 2003

Sponsored by

The Winnipeg Actuaries Club

The Manitoba Association of Mathematics Teachers

The Canadian Mathematical Society

and

The University of Manitoba

ANSWER AS MUCH AS POSSIBLE. YOU ARE NOT EXPECTED TO COMPLETE THE PAPER. SEE BOTH SIDES OF THIS SHEET. HAND CALCULATORS ARE NOT PERMITTED. NUMERICAL ANSWERS ONLY, WITHOUT EXPLANATION, WILL NOT BE GIVEN FULL CREDIT.

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1. (a) Solve the equation $\frac{1}{x} + \frac{1}{x+2} = \frac{1}{x^2 + 2x}$.

   (b) If a and b are non-zero real numbers such that $9a^2 - 12ab + 4b^2 = 0$, find the numerical value of $\frac{a}{b}$.

2. (a) Today Joe's son is $\frac{1}{3}$ of Joe's age. Five years ago he was $\frac{1}{4}$ of Joe's age at that time. How old is Joe's son now?

   (b) a and b are non-zero real numbers. If the equation $ax^2 + bx + 8 = 0$ has exactly one solution, find the numerical value of $\frac{b^2}{a}$.

3. (a) A rectangle is twice as long as it is wide and has a diagonal of length 5. What is its area?

   (b) If the perimeter of an isosceles right-angled triangle is 8, what is its area?

(See other side)

4. (a) Find the length of the diameter of a circle whose area is tripled when the length of its radius is increased by 2.

   (b) If a and b are real numbers such that $3(2^a) + 2^b = 7\sqrt{2}$ and $5(2^a) - 2^b = 9\sqrt{2}$, find a and b.

5. (a) If $\sec\theta + 9\cos\theta = 6$, what is the numerical value of $\sec\theta$?

   (b) The point A lies on the line whose equation is $y = x$. The point B lies on the line whose equation is $y = -x$. The line segment AB has length 2. Prove that the mid-point of the line segment AB lies on a circle of radius 1 with centre at the origin.

6. (a) In this problem O is the origin, A is the point (3, 1) and P is a point in the first quadrant on the graph of $3x - 4y = 0$. If $\angle APO = 45°$ find the area of triangle AOP.

   (b) If r, s, and t are real numbers such that $r - 2s + 3t \geq 2$ and $2r + s - 3t \geq 1$ prove that $7r - 4s + 3t \geq 8$.

7. A right-angled triangle has area 5. The altitude perpendicular to the hypotenuse has length 2. Find the lengths of the three sides of the triangle.

8. A and B are points on the graph of the equation $(x^2 + y^2 - 1)\{(x-1)^2 + (y-1)^2 - 2\} = 0$.
   What is the largest possible value for the length of the line segment AB? Prove that your answer is correct.

9. In this problem O is the origin. P is a point on the graph of $y = x^2$. The coordinates of P are non-zero integers. Prove that the length of the line segment OP cannot be an integer.

10. (a) Solve the equation $x^4 - 6x^2 + 9 = (x+1)^2$.

    (b) Solve the equation $x^4 - 7x^2 = 4x - 5$.