1. A function $f : \mathbb{Z} \to \mathbb{Z}$ is defined by

$$f(n) = \begin{cases} n-1 & \text{if } n \text{ is odd,} \\ n^2-1 & \text{if } n \text{ is even.} \end{cases}$$

   Determine all integers n for which $f(f(n)) = 63$.

2. Determine all solutions for $x, y \in \mathbb{R}$ to the equations

$$\begin{aligned} 2x + yx - 16x^3 &= 0 \\ y + 6xy - 2y^2 &= 0. \end{aligned}$$

3. Suppose the sequence $x_1, x_2, \ldots$ is an arithmetic progression with $x_1 \neq x_2$ with the property that x_1, x_2, x_{2016} is a geometric progression. Determine the set of indices (i, j) $(i < j)$ for which x_1, x_i, x_j is also a geometric progression.

4. Determine the value of $x + y$ if

$$\frac{(x-2015)^2 + (y-2016)^2}{(x-2015)(y-2016)} = -2.$$

5. Determine all real numbers x for which

$$(\ln x)^{\ln(\ln x)} = 2016.$$

6. Determine all linear functions $f : \mathbb{R} \to \mathbb{R}$ with the property that for all x,

$$f(x) - f^{-1}(x) = 2016.$$

   (Recall that a linear function is of the form $f(x) = ax + b$.)

7. Determine all positive integers a, b with the property that

$$a^3 + 4ab = 20437.$$

8. The function $f : \mathbb{Z} \to \mathbb{Z}$ has the property that for all x,

$$f(x) = f(x-1) + f(x+1).$$

   If $f(0) = 1$ and $f(1) = 3$, what is the value of $f(2016)$?

9. If $x \in \mathbb{R}$ with $x^3 + \frac{1}{x^3} = 2$, what are the possible values for $x^2 + \frac{1}{x^2}$?