HW #4

Li

#1 Here we want to find a formula for a solution

$u = u(x,t)$

$$\begin{cases} u_{tt} = u_{xx} & x \in \mathbb{R},\ t > 0 \\ u(x,0) = \phi(x) & x \in \mathbb{R} \\ u_t(x,0) = \psi(x) & x \in \mathbb{R}. \end{cases}$$

(i) show $u(x,t) = F(x+t) + G(x-t)$

is a solution of

$u_{tt} = u_{xx} \quad x \in \mathbb{R},\ t > 0$

For any differentiable

F & G.

(ii) Try & find F & G such that you solve the full problem.

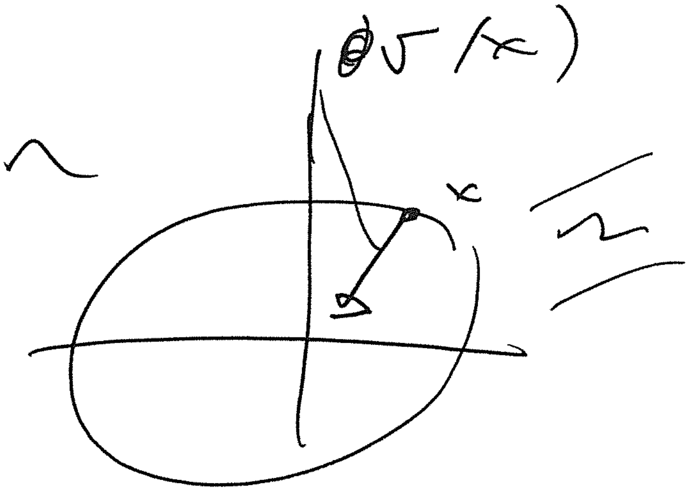
$\boxed{\#2}$

let $\Omega = \{x \in \mathbb{R}^2 : |x| > 1\}$.

let $\alpha > 0$ & consider solving

$$\begin{cases} \Delta u = 0 & x \in \Omega \\ u + \alpha \partial_\nu u = 0 & \partial\Omega \\ \displaystyle\lim_{|x|\to\infty} \frac{u(x)}{\ln(|x|)} = 7. & \end{cases}$$

<u>Note</u> for $x \in \partial\Omega$



$\boxed{\#3}$

Assume

$$-\frac{d}{dx}\left(p(x)\,\phi_n'(x)\right) = \lambda_n \phi_n(x) \qquad x \in (0,\pi),$$

$$\phi_n(0) = \phi_n(\pi) = 0, \qquad n \geq 1.$$

From earlier we know for $\lambda_n \neq \lambda_m$ that $\int_0^{\pi} \phi_n(x)\,\phi_m(x)\,dx = 0.$

(i)

Assume we can write any $f(x)$ by

$$f(x) = \sum_{n=1}^{\infty} a_n \phi_n(x), \quad a_n \in \mathbb{R}.$$

Find a formula for $a_n(t)$. [5

(ii) Now find a solution of $u = u(x,t)$

$$
\begin{cases}
u_t = \dfrac{\partial}{\partial x}\left(p(x)\, u_x(x,t)\right) & x \in (0,\pi) \\
 & t > 0 \\
u(0,t) = u(\pi,t) = 0 & t > 0 \\
u(x,0) = \phi(x) & x \in (0,\pi).
\end{cases}
$$
