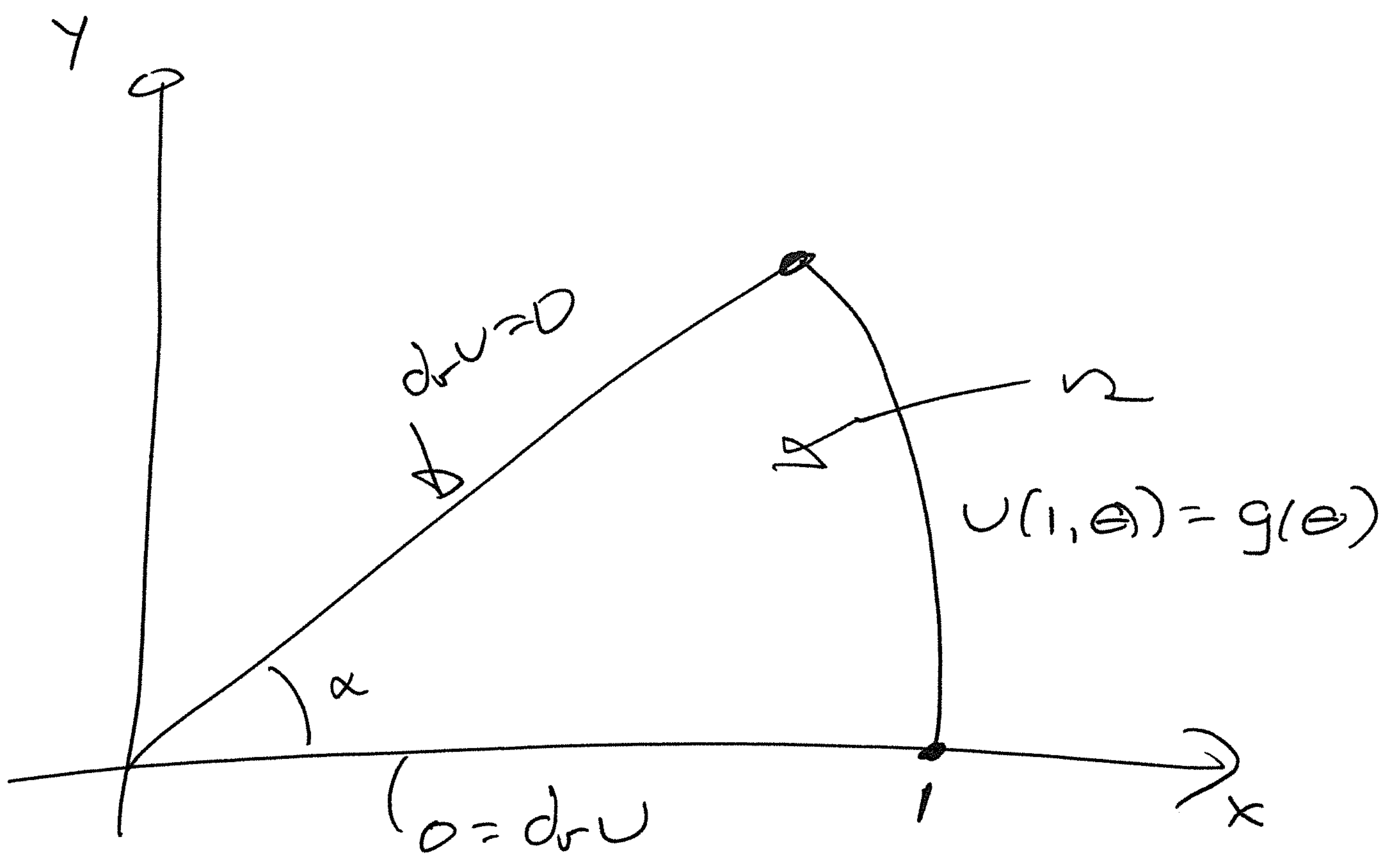
4

Homework 3

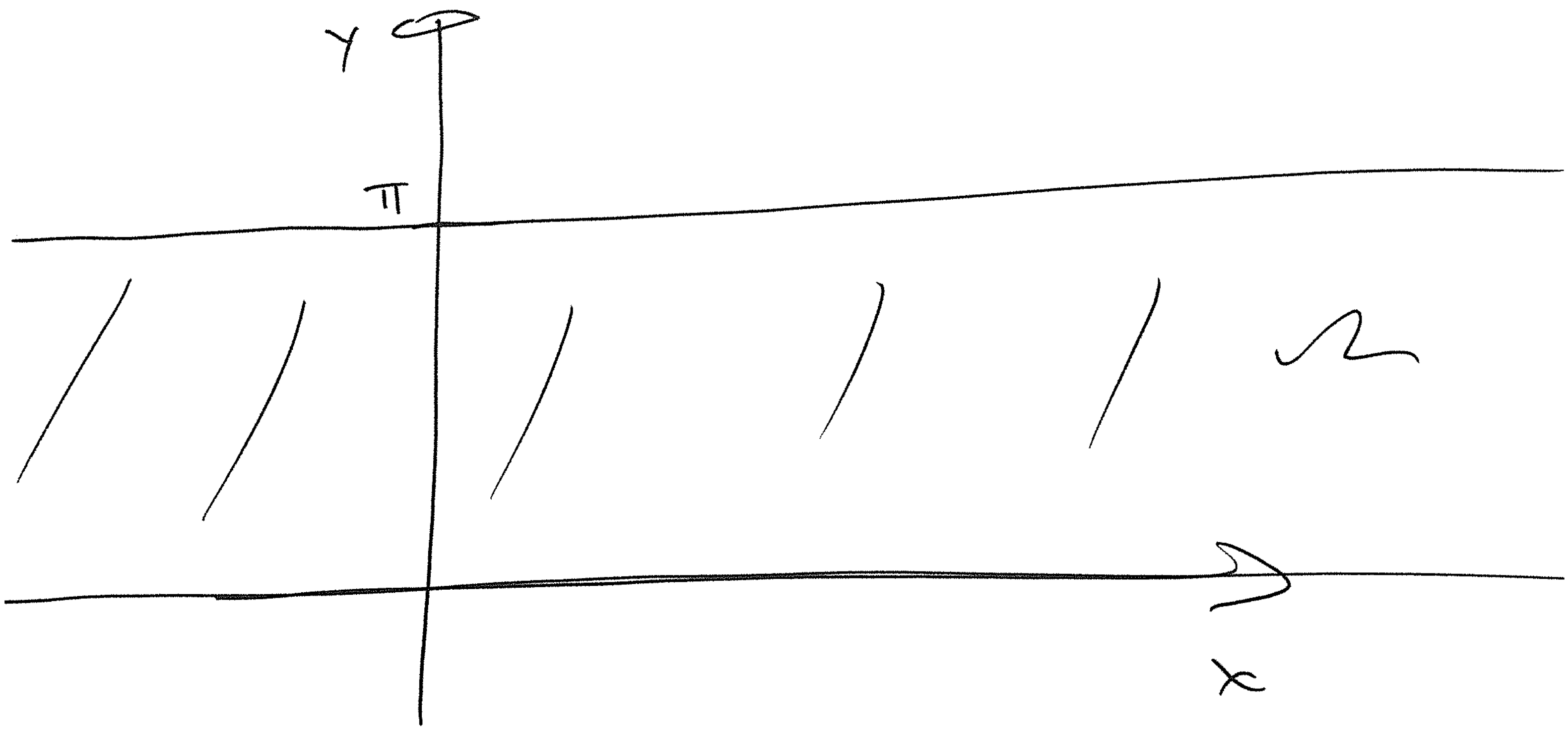
$\boxed{\#1}$ Solve $u = u(x,y)$

$$\begin{cases} \Delta u = 0 \text{ in } \Omega \end{cases}$$



$\boxed{\#2}$

Let $\Omega = \left\{ (x,y) \in \mathbb{R}^2 : \begin{array}{c} x \in \mathbb{R} \\ 0 < y < \pi \end{array} \right\}$.



Question is about maximum principle on unbounded domains.

Show maximum principle fails on Ω.

Find a u s.t.

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

but $u \neq 0$ in Ω.

Hint Try seperation of variables.

#3 let $u = u(x,t)$ solve

$$\begin{cases} u_{tt} - u_{xx} = -u^3 & (x,t) \in (0,\pi) \times (0,\infty) \\ u(x,0) = \varphi(x) \\ u_t(x,0) = \psi(x) \\ u(0,t) = u(\pi,t) = 0. \end{cases}$$

Set

$$E(t) = \frac{1}{2}\int_0^\pi u_x^2 + u_t^2 \, dx + \frac{1}{4}\int_0^\pi u^4 \, dx.$$

show E a conserved quantity

ie $E'(t) = 0$.

5

$\boxed{\#4}$ let $0 < \alpha < \frac{\pi}{2}$

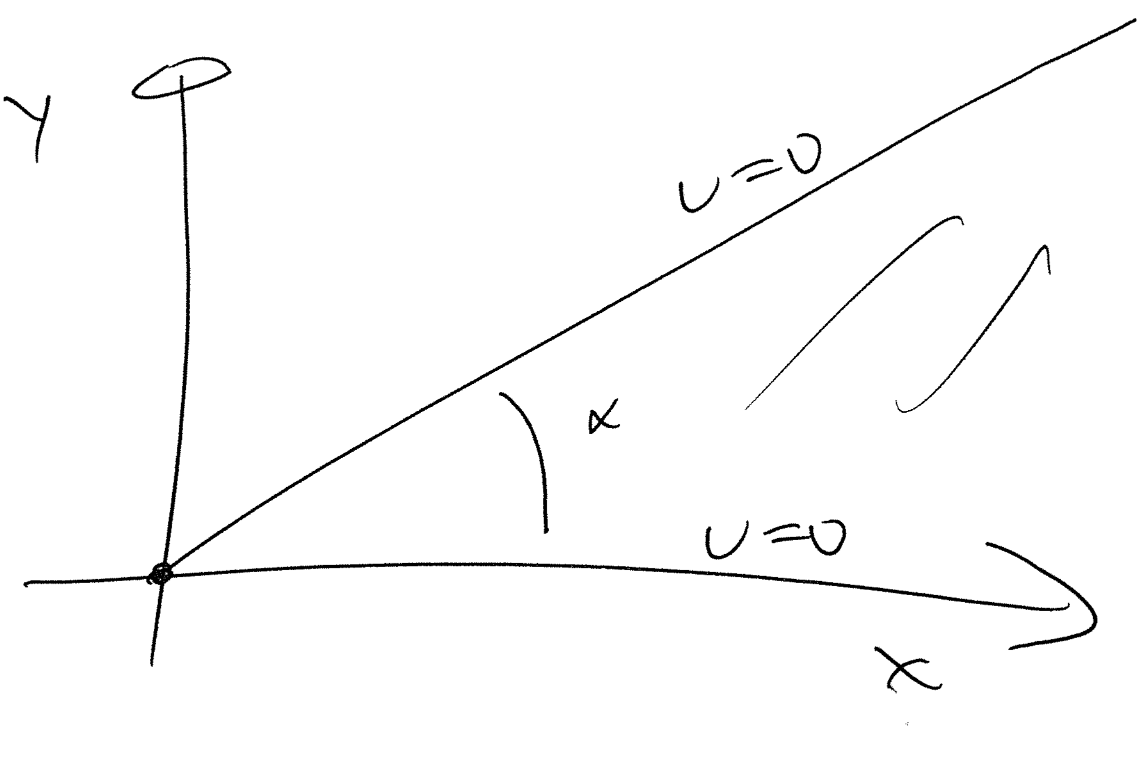
consider

$\Omega = \left\{ (x,y) \in \mathbb{R}^2 : \begin{array}{c} \text{in polar} \\ 0 < \theta < \alpha \\ r > 0 \end{array} \right\}$

Let $1 < p$. Goal Find positive solution

of

$$\begin{cases} -\Delta u(x,y) = u(x,y)^p & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \setminus (0,0) \end{cases}$$



Try looking for a solution of form

$u(r,\theta) = r^t w(\theta)$.