HW #1 | Due Wed Sept 19 | L1

#1 show $d(x,y) = |x-y|$

is a metric on $\mathbb{R}$.

#2 show $e(x,y) = |x-y| + \left|\frac{1}{x} - \frac{1}{y}\right|$

A metric on $\mathbb{R}$.

(3) (2)

#3 Let $I = [0,1]$

let $F := \{ f \in C(I) : f(0) = 0$

$|f(x) - f(y)| \leq 10|x-y|$

$\forall x, y \in I. \}$

show F is a compact subset of $C(I)$.

Hint let $(f_n)_n \subset F$. show $\exists f \in F$ s.t. $f_n \to f$ in $C(I)$.

#4 Consider $\mathbb{R}$ with the discrete metric

$$d(x,y) = \begin{cases} 0 & x = y \\ 1 & x \neq y. \end{cases}$$

(i) Show $(\mathbb{R}, d)$ a metric space.

(ii) Is $[0,1]$ open, closed & (or) neither in $(\mathbb{R}, d)$?