Ex $f(x) = \frac{x+1}{x-2}$, $g(x) = \sqrt{x+1}$.

compute $f \circ g$ & $g \circ f$, without computing domain

① $(f \circ g)(x) = f[g(x)]$

$$= \frac{g(x)+1}{g(x)-2} = \frac{\sqrt{x+1}+1}{\sqrt{x+1}-2}$$

or we could have

$$f[g(x)] = f[\sqrt{x+1}] = \frac{\sqrt{x+1}+1}{\sqrt{x+1}-2}$$

---

② $(g \circ f)(x) = g(f(x))$

$$= \sqrt{f(x)+1}$$

$$= \sqrt{\frac{x+1}{x-2}+1}$$

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compute & find domains

a) $f \circ g$

b) $g \circ f$, $\quad f(x) = \sqrt{x}$, & $g(x) = \sqrt{2-x}$.

(a) $(f \circ g)(x) = f[g(x)]$

$= \sqrt{g(x)} = \sqrt{\sqrt{2-x}}$

$= \sqrt[4]{2-x} = (2-x)^{\frac{1}{4}}$

need $2-x \geq 0$ or $x \leq 2$

or $x \in (-\infty, 2]$ or

$(-\infty, 2]$.

(b) $(g \circ f)(x) = g[f(x)]$

$= g[\sqrt{x}] = \sqrt{2-\sqrt{x}}$

need $x \geq 0$ (so $\sqrt{x}$ makes sense) i need $2-\sqrt{x} \geqslant 0$

$\Rightarrow 2 \geqslant \sqrt{x} \Rightarrow \boxed{x \leq 4}$

so domain is $[0, 4]$.

.4 Exponential Functions [

Recall: b^{x} ← exponent.
b = base

An exponential function is a function like

$f(x) = a^x \quad a > 0$ (note x is exponent, not x^a ← power function!)

If $n \geq 1$ an integer

$a^n = \underbrace{a \cdots a}_{n \text{ times}}$

~~For Bt a fraction~~

For $\frac{m}{n}$ a fraction

$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$ or $= (a^{\frac{1}{n}})^m \quad a > 0.$

aw of exponents

$$a^{x+y} = a^x a^y$$

$$a^{x-y} = \frac{a^x}{a^y}$$

$$(a^x)^y = a^{xy}.$$

$$a^{-x} = \frac{1}{a^x}.$$

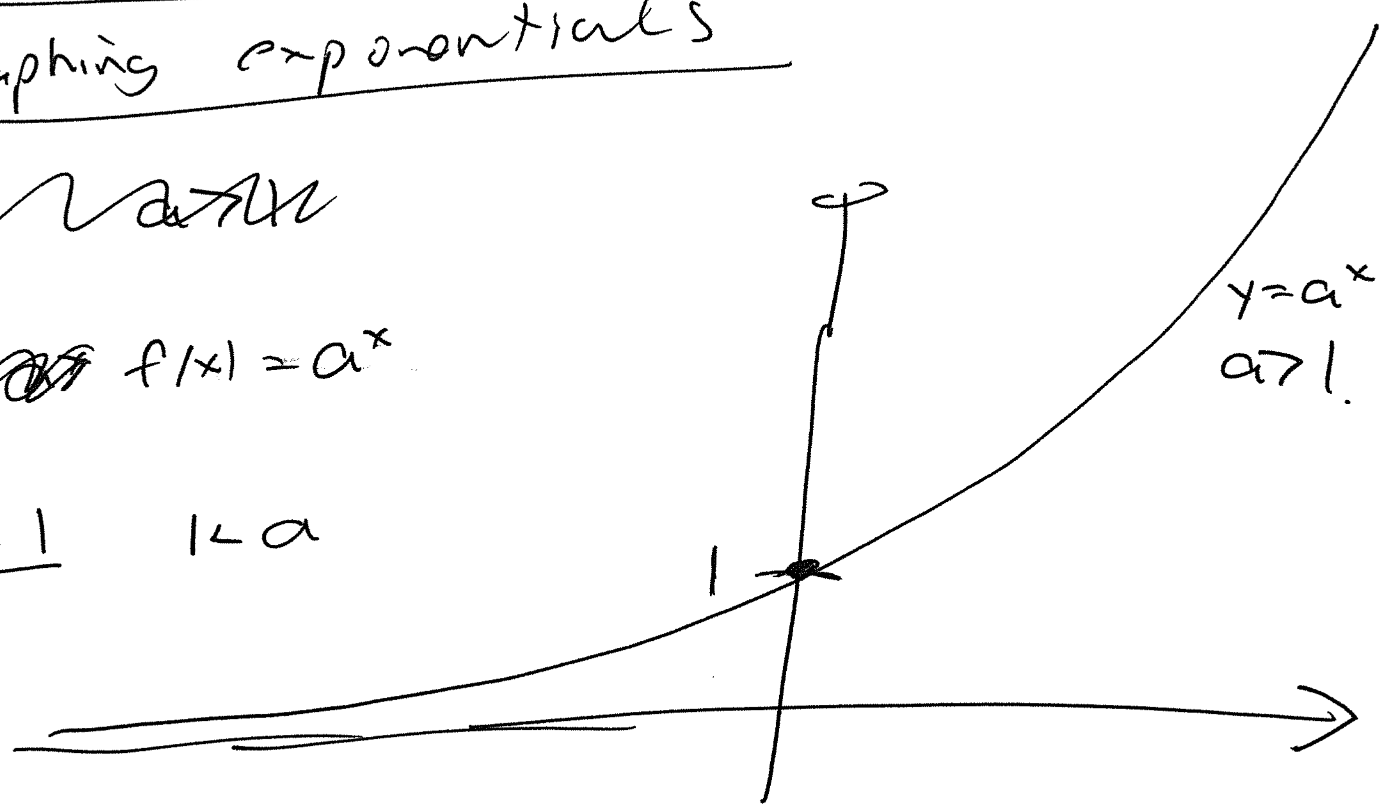
$$a^0 = 1.$$

Graphing exponentials

~~For a>1~~

$f(x) = a^x$

case 1 $\quad 1 < a$



Since $a > 1$ we know

a^n big for n Big.

For $0 < a < 1$ note

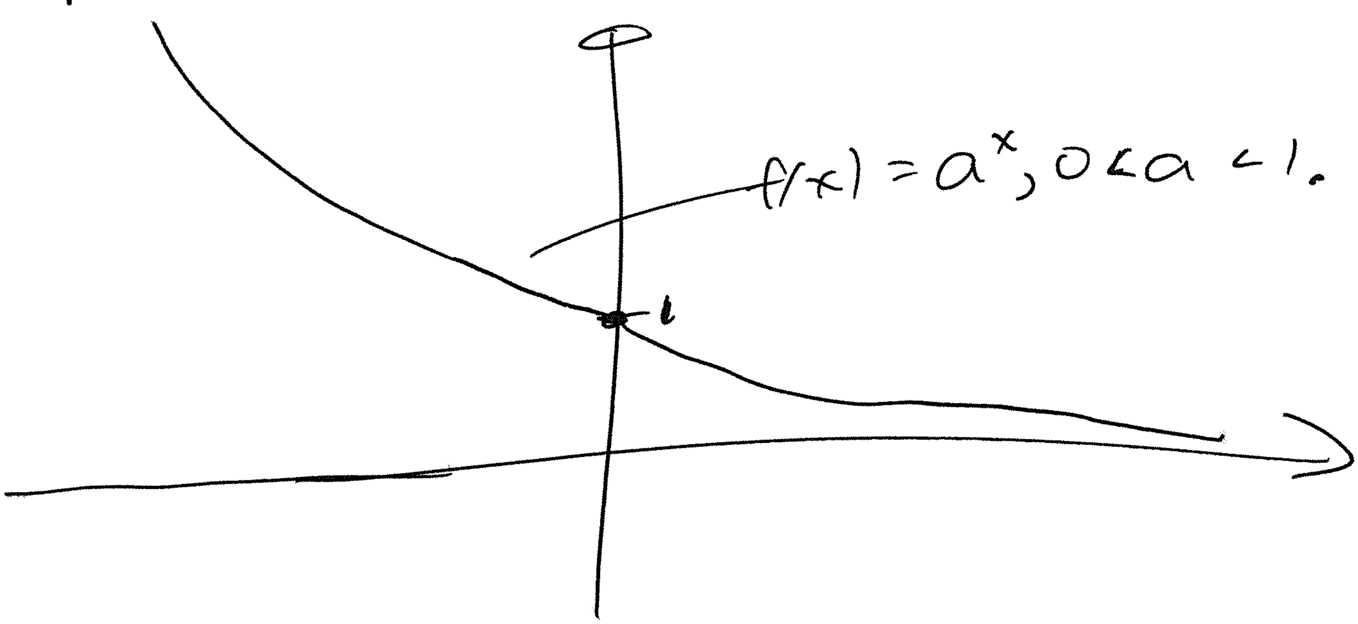
$$f(x) = a^x = \left(\left(\frac{1}{a}\right)^{-1}\right)^x = \left(\frac{1}{a}\right)^{-x}$$

& note $\frac{1}{a} > 1$ so expect

graph to have same shape

but since its $-x$ not x

its reflected in y-axis.



For our exponential functions a^x we will almost always use $a=e$ where

$e \approx 2.71828\ldots$

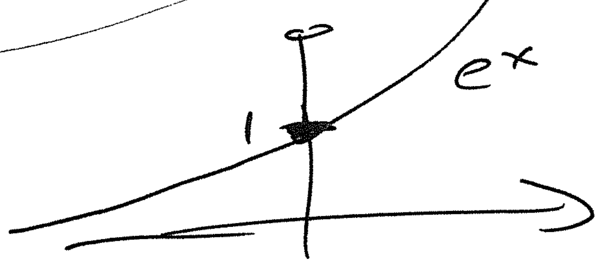
Very special # in calculus.

~~So for~~ $f(x)=e^x$ we will use all the time.

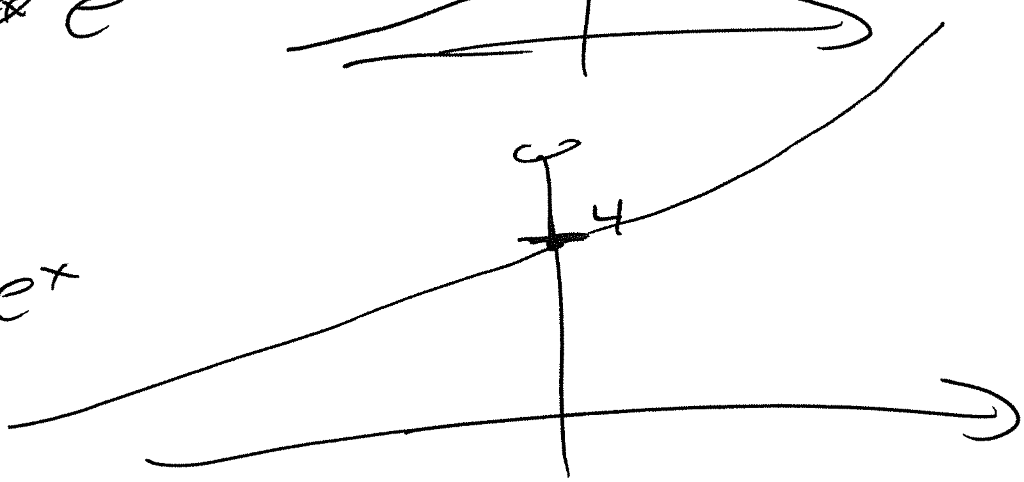
Ex Graph $y=f(x)=4e^x-5$, & find domain & range

since $e>1$.

e^x

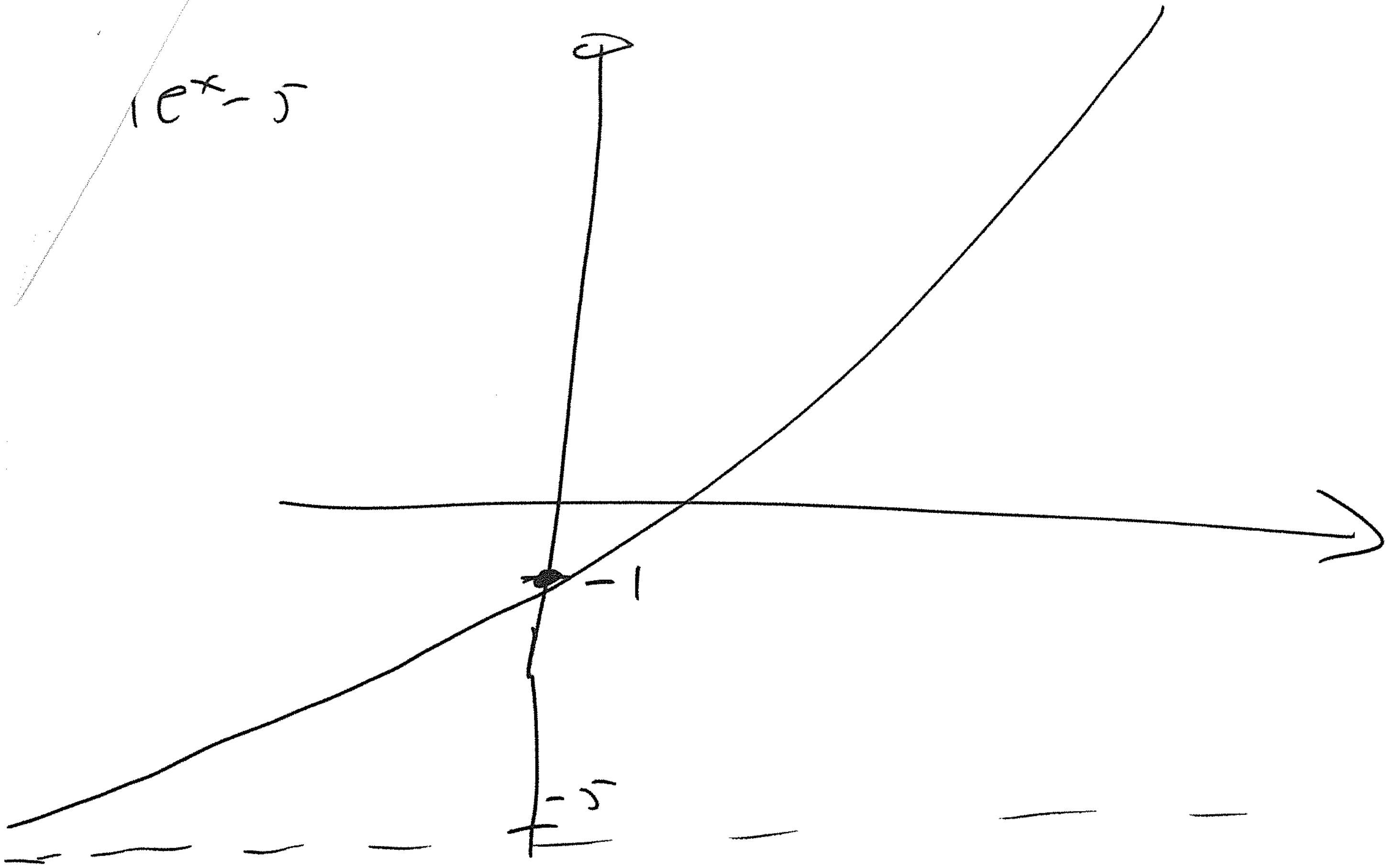


$4e^x$



$4e^x - 5$

8



domain of $4e^x - 5 = \mathbb{R}$.

range $= (-5, \infty)$.

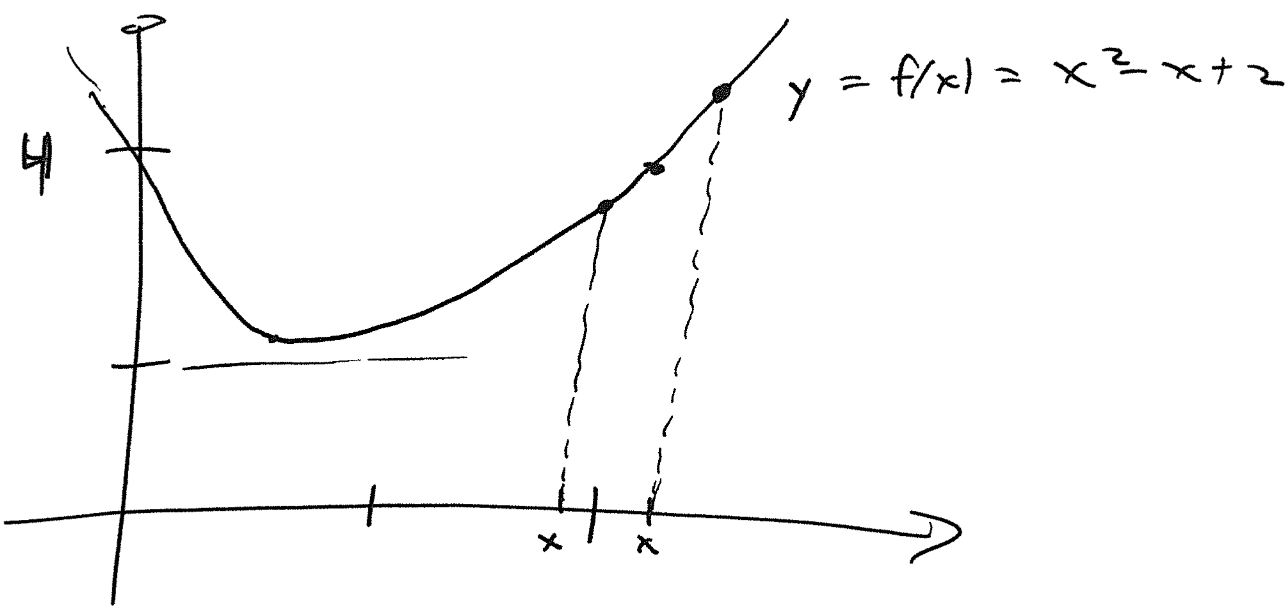
(maybe Add more)

ection 2.2 Limit of a function (whole course is built on limits)

consider

$f(x) = x^2 - x + 2$ for values of x close to $x = 2$.

If we graph



x	$f(x)$
1.9	3.7100
1.95	3.85525
1.999	3.970100
2.01	4.03
2.005	4.015025
2.001	4.003001

Note as x gets closer to 2 that $f(x)$ gets closer ~~to~~ 4.
to
& closer

we write

$$\lim_{x \to 2} x^2 - x + 2 = 4.$$

In general, if $f(x)$ is a function that gets closer & closer to a # L as x gets closer & closer to some # a then we write

$\lim_{x \to a} f(x) = L$.

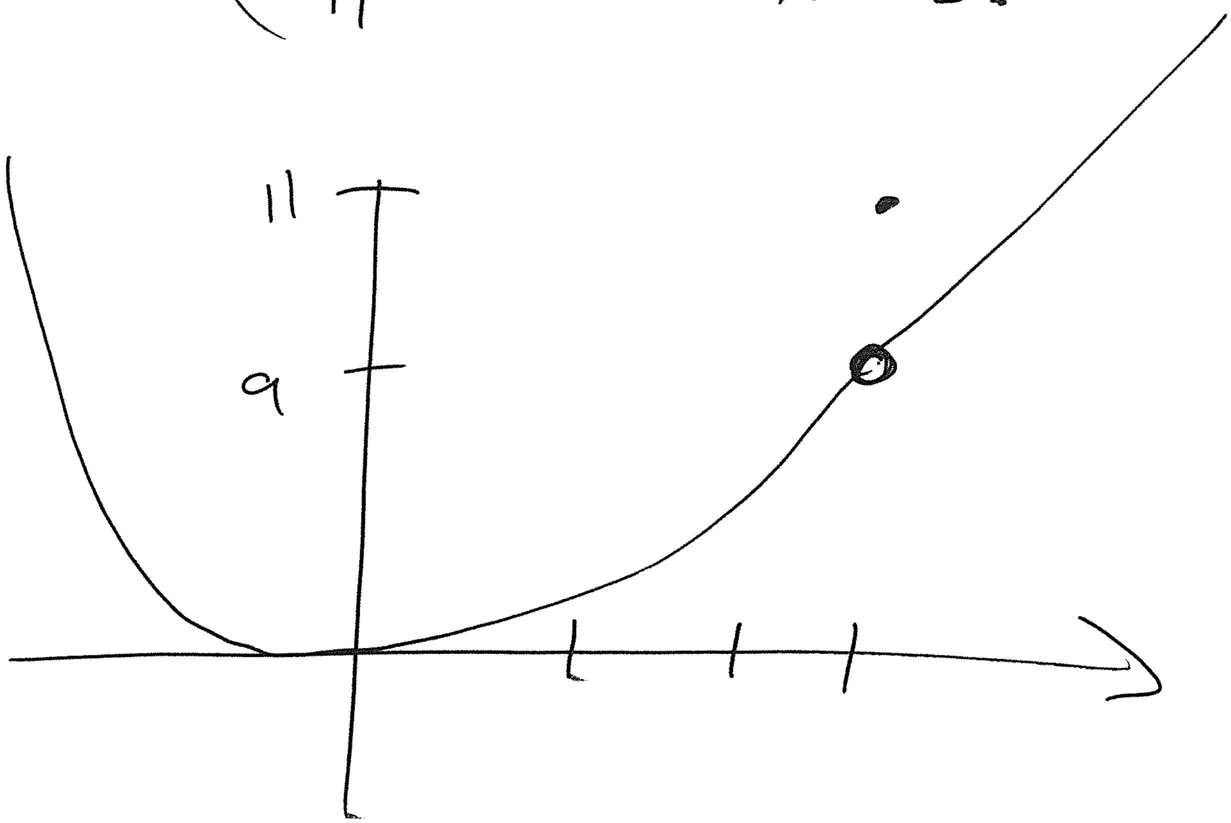
$\boxed{\text{or you can write } f(x) \to L \text{ as } x \to a.}$

<u>Note</u>: The above must work for x bigger & smaller than a (but close).

We never plug in $x = a$ & we don't care about f(a) or whether f(a) even defined.

consider

$$f(x) = \begin{cases} x^2 & x \neq 3 \\ 11 & x = 3. \end{cases}$$



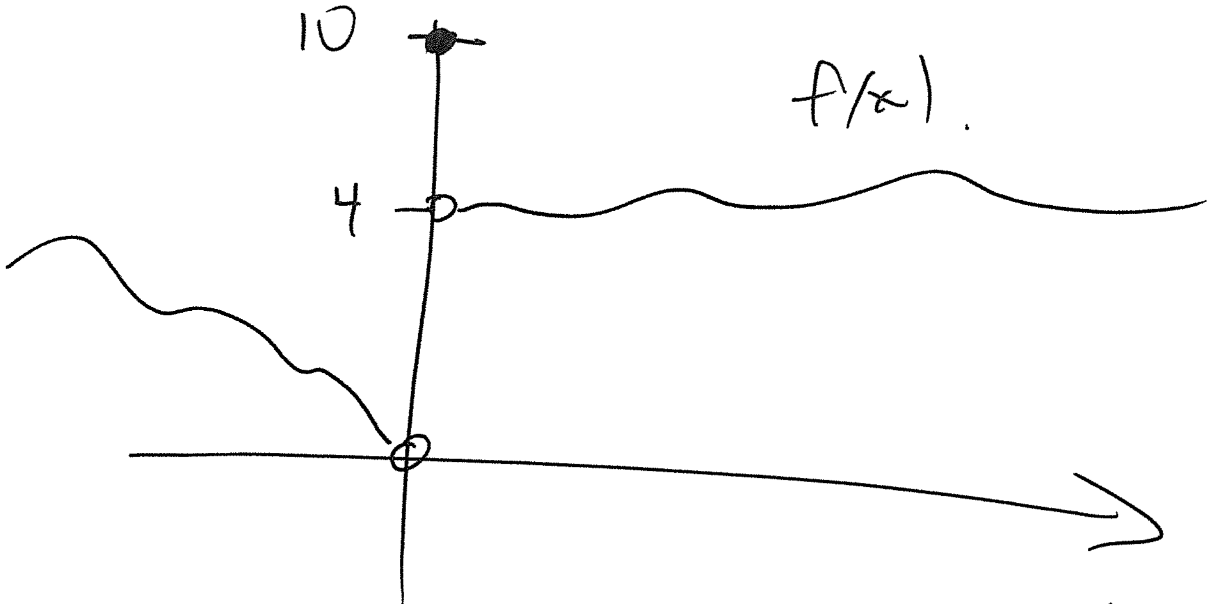
Find $\lim_{x \to 3} f(x) = ?$.

-un x close to $x=3$ but $x \neq 3$ $f(x)$ gets closer & closer to 9; so

$$\lim_{x \to 3} f(x) = 9.$$

note $f(3)=11$ is irrelevent to $\lim_{x \to 3} f(x)$

Ex Consider $f(x)$ whose graph is given by



Compute $\lim_{x \to 0} f(x)$ & $f(0)$.

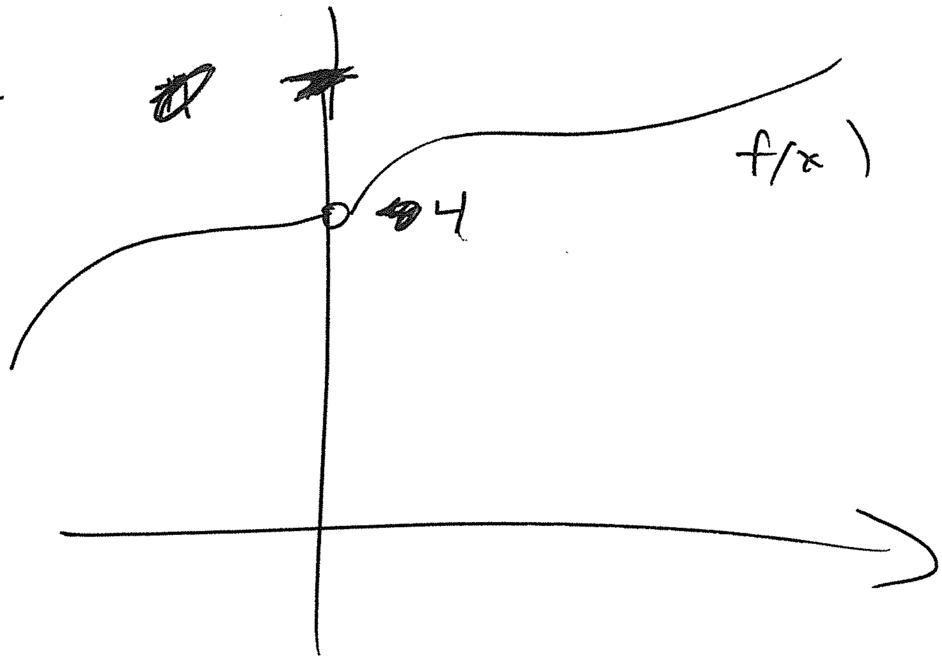
$f(0) = 10$.

As x gets close to zero from the right (left) f(x) gets close to 4 (0)

& note $4 \neq 0$, so

$\lim_{x \to 0} f(x)$ does not exist.

Ex



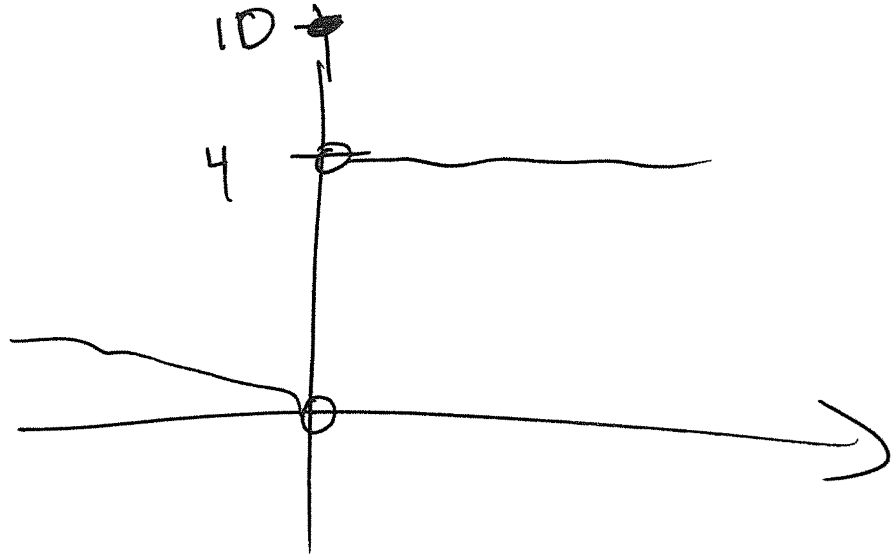
Find $f(0)$ & $\lim_{x \to 0} f(x)$.

Solution $f(0)$ is not defined,

& $\lim_{x \to 0} f(x) = 4$.

one sided limits

consider f(x) from 2 examples ago



$\lim_{x \to 0} f(x)$ does not exist.

but we can talk about one sided limits.

~~left hand limit~~

$\lim_{x \to 0^-} f(x) = 0$

As x gets close to x=0 from left f(x) gets close to 0.

~~right hand~~

$\lim_{x \to 0^+} f(x) = 4$

As x gets closer to x=0 from right f(x) gets close to 4.

$\lim_{x \to 0^-}$ (limit at 0 from left)

$\lim_{x \to 0^+}$ (limit at 0 from right).

<u>Thm</u>

$\lim_{x \to a} f(x) = L$ if & only if $\lim_{x \to a^+} f(x) = L$

& $\lim_{x \to a^-} f(x) = L$

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Ex Consider $f(x) = \dfrac{x^2+2x+1}{x+1}$.

Note bottom $= 0$ zero at $x=-1$

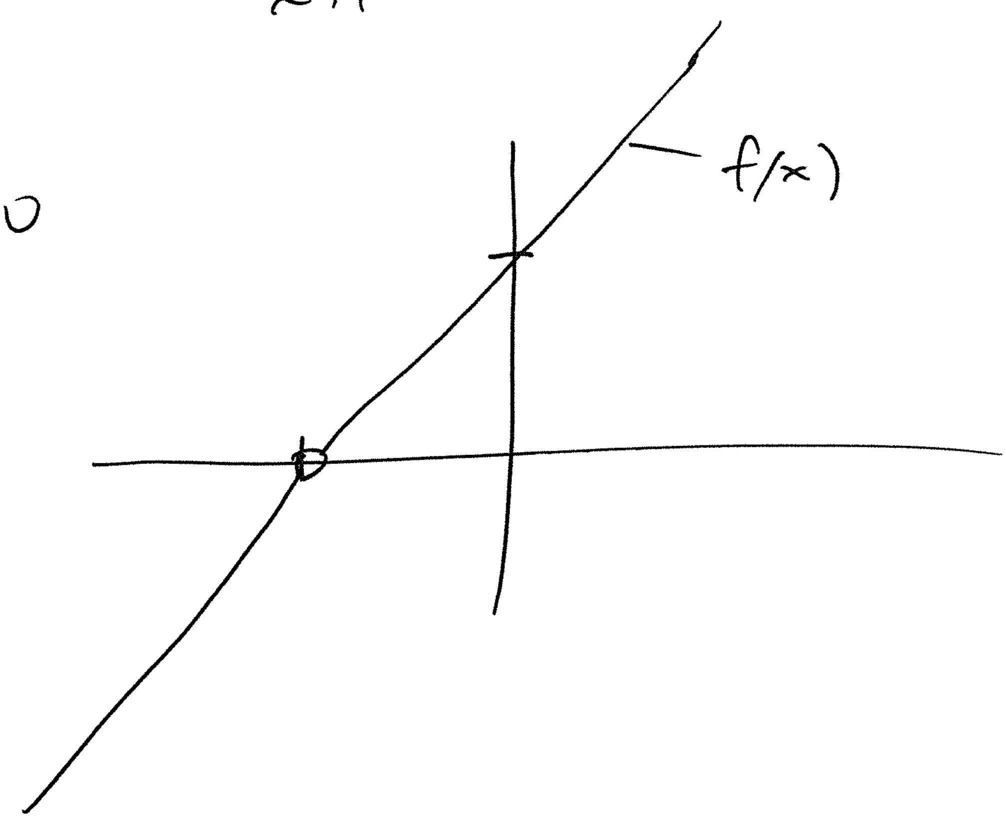
so $x=-1$ not in domain of f.

Compute $\lim_{x\to 3} f(x)$ & $\lim_{x\to -1} f(x)$.

Solution.

$$f(x) = \frac{x^2+2x+1}{x+1} = \frac{(x+1)^2}{x+1} = x+1 \quad \text{provided } x \neq -1.$$

So

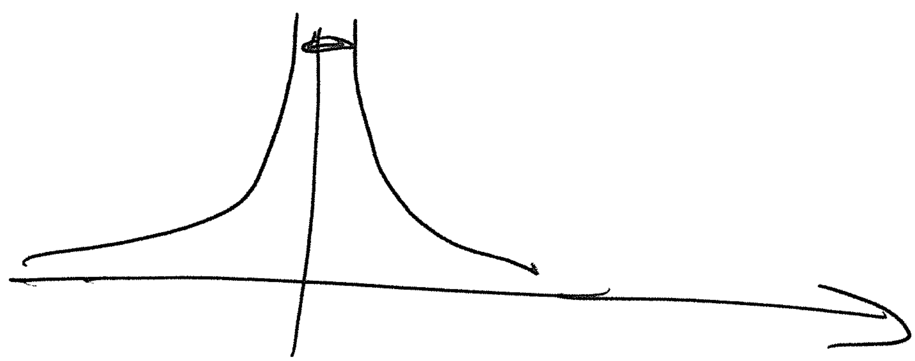


so $\lim_{x \to 3} f(x) = \lim_{x \to 3} (x+1) = 4.$

$\lim_{x \to -1} f(x) = \lim_{x \to -1} (x+1) = 0.$

Infinite limits

Consider $f(x) = \frac{1}{x^2}$



$\lim_{x \to 0} \frac{1}{x^2}$ Does not exist.

As x gets close to zero

$f(x)$ does not get close

to some # (recall ∞ is not a number).

so strictly speaking

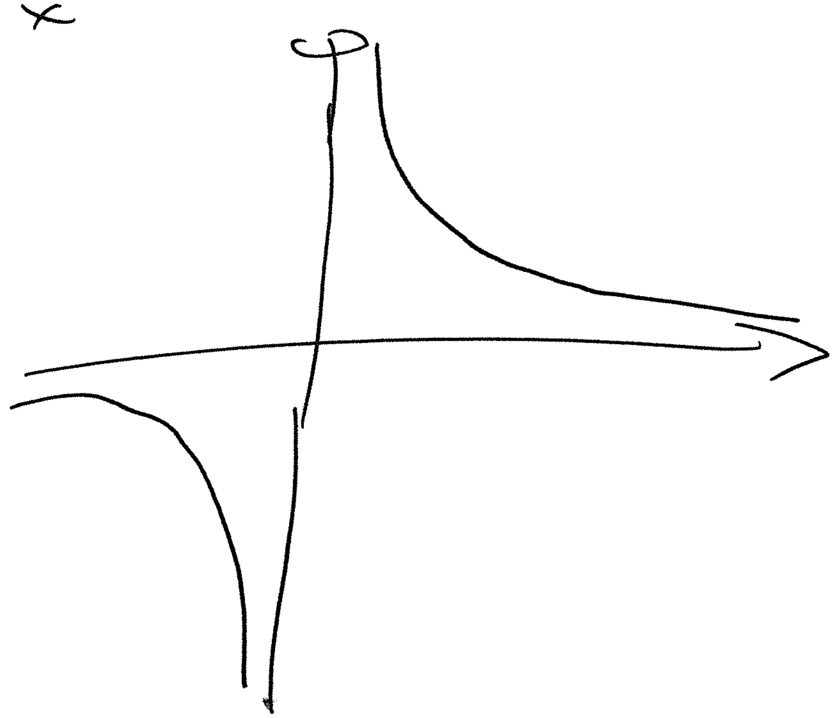
$\lim_{x \to 0} \frac{1}{x^2}$ does not exist;

but in some sense saying

$\lim_{x \to 0} \frac{1}{x^2} = \infty$ does give us

some information.

Ex $\lim_{x \to 0} \frac{1}{x}$ does not exist



But we can say

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty.$$

__Defn.__ The vertical line $x = a$ is a __vertical Asymptote__ of the graph $y = f(x)$ if at least one of the following Holds:

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty$$

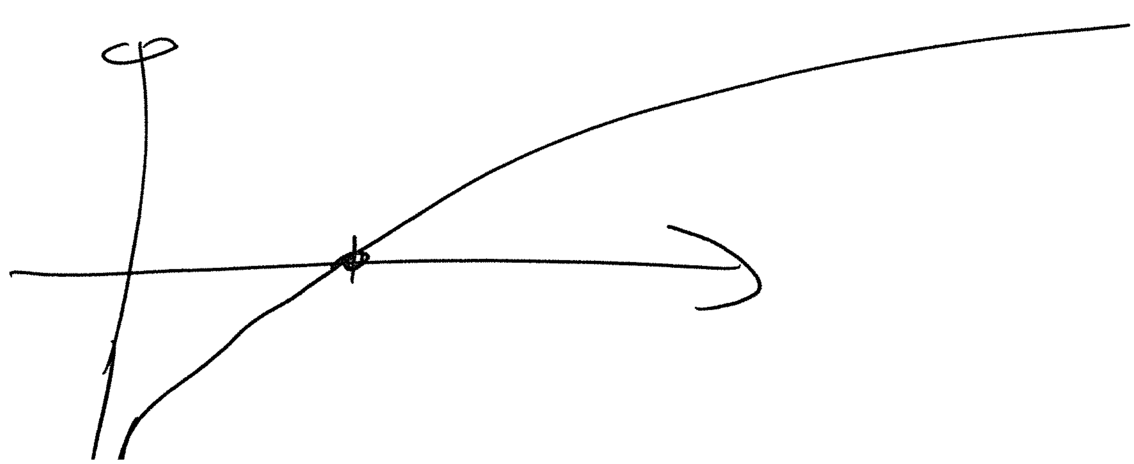
$$\lim_{x \to a^+} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = -\infty.$$

Ex Find all vertical Asymptotes of $f(x) = \frac{2x}{x-3}$.

$\lim_{x \to 3^+} f(x) = \infty \therefore$ The line $x = 3$ is a vertical Asymptote.

(note: 3 is not vertical Asymptote; its the line $x = 3$)

Ex Recall $\ln(x) = f(x)$ natural logarythm is



so $\lim_{x \to 0^+} \ln(x) = -\infty$

so $x = 0$ is a vertical Asymptote for the graph.

Section 2.3 Limit Laws.

Limit laws

If $c \in \mathbb{R}$ (ie c constant)

& $\lim_{x \to a} f(x) = L_1$ & $\lim_{x \to a} g(x) = L_2$

then

1. $\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

$= L_1 \pm L_2.$

2. $\lim_{x \to a} (c\, f(x)) = c \lim_{x \to a} f(x) = c L_1$

(3) $\lim\limits_{x \to a} \left(f(x) g(x) \right) = \left(\lim\limits_{x \to a} f(x) \right) \left(\lim\limits_{x \to a} g(x) \right)$

$= L_1 L_2$

(4) If $L_2 \neq 0$ then

$$\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} = \frac{L_1}{L_2} .$$

Ex Suppose $\lim\limits_{x \to 1} f(x) = 4$, $\lim\limits_{x \to 1} g(x) = -$

~~T~~ calculate $\lim\limits_{x \to 1} \left(5 f(x) - 2 f(x) g(x) \right)$.

solution

$$\lim_{x \to 1} \left(5 f(x) - 2 f(x) g(x) \right)$$

$$= \lim_{x \to 1} \left(5 f(x) \right) - \lim_{x \to 1} \left(2 f(x) g(x) \right) \quad \left(\text{Law } 1 \right)$$

$$= 5 \left(\lim_{x \to 1} f(x) \right) - 2 \left(\lim_{x \to 1} f(x) g(x) \right) \quad \left(\text{Law } 2 \right)$$

$$= 5 \left(\lim_{x \to 1} f(x) \right) - 2 \left(\lim_{x \to 1} f(x) \right) \left(\lim_{x \to 1} g(x) \right) \quad \left(\text{Law } 3 \right)$$

$$= 5 (4) - 2 (4)(-2)$$

$$= 20 + 16 = 36.$$

More laws (there are clear from graph)

(5) $\lim_{x \to a} c = c$

(6) $\lim_{x \to a} x^p = a^p$ (p Any power)

As long as a^p is defined.

Ex Calculate $\lim_{x \to 2} \dfrac{x^2+x-1}{3x+5}$

(using the limit laws).

<u>solution</u>

$$\lim_{x\to 2}\left(\frac{x^2+x-1}{3x+5}\right)$$

$$= \frac{\lim_{x\to 2}(x^2+x-1)}{\lim_{x\to 2}(3x+5)}$$

use $\lim_{x\to 2} 3x+5 \neq 0$

$$= \frac{\lim_{x\to 2}(x^2) + \lim_{x\to 2}(x) + \lim_{x\to 2}(-1)}{\lim_{x\to 2}(3x) + \lim_{x\to 2}(5)}$$

$$= \frac{\lim_{x\to 2}(x^2) + \lim_{x\to 2}(x) + \lim_{x\to 2}(-1)}{3\lim_{x\to 2}(x) + \lim_{x\to 2}(5)}$$

$$= \frac{4+2-1}{3(2)+5} = \frac{5}{11}.$$